BNAD/ECON/MGMT 276
Lecture 5
Discrete Probability Distributions
Exercises

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Outline

Exercise 1

Suppose we have the probability distribution for the random variable X as follows.

X	$f(x)$
20	.20
25	.15
30	.25
35	.40

- Is this probability distribution valid? Explain.
- What is the probability that $X = 30$.
- What is the probability that X is less than or equal to 25?
- What is the probability that X is greater than 30?
- What is the expected value (or mean) of X?
- What is the variance of X?

Exercise 2

A psychologist determined that the number of sessions required to obtain the trust of a new patient is either 1, 2, or 3. Let X be a random variable indicating the number of sessions required to gain the patient's trust. The following probability function has been proposed.

$$f(x) = \frac{x}{6} \quad \text{for } x = 1, 2, \text{ or } 3.$$

a. Is this probability function valid? Explain.

b. What is the probability that it takes exactly two sessions to gain the patient's trust?

c. What is the probability that at least two sessions to gain the patient's trust?

Exercise 3

There is a random variable X which has the following probability function.

X	$f(x)$	$F(x)$
1	0.5	
2	0.2	
3	0.2	
4	0.05	
5	0.05	

- Construct the cumulative probability function, $F(x)$, in the above table.
- Calculate $E(X)$ and $Var(X)$.

Exercise 3 Cont'd

Now we are interested in a random variable $Y = 1 + 2X$.

- What is the probability that $Y = 11$?
- Calculate $E(Y)$ and $Var(Y)$

Exercise 4: A Bernoullie Process

Consider the probability function for a random variable X in the following. 1 stands for *Success* and 0 stands for *Failure*.

X	$f(x)$
0	0.4
1	0.6

- Calculate the expected value (mean) of X.
- What is the probability of *Success*?
- Calculate the variance of X.

Exercise 5

a) consider the following experiment.

We toss a coin, roll a die, and toss a coin again. Is this experiment a binomial experiment? Explain.

b) consider the following experiment.

We have a special coin that has the following property. Once a coin landed with $H(T)$, the probability that we have $H(T)$ in the next time increases. With this coin we conduct the experiment of tossing this coin 3 times. Is this experiment a binomial experiment? Explain.

Exercise 6

Consider a binomial experiment with $n = 10$ and $p = 0.10$.

a. Compute $f(0)$

b. Compute $f(2)$

c. Compute $P(X \leq 2)$.

d. Compute $P(X \geq 1)$.

e. Compute $E(X)$

f. Compute $Var(X)$.

Exercise 6 cont'd

d. What is the mean of X?

e. What is the variance and standard deviation of X?

Exercise 7

In San Francisco, 30% of worker take public transportation daily (*USA Today, December 21, 2005*).

a. In a sample of 10 workers, what is the probability that exactly three workers take public transportation?

b. In a sample of 10 workers, what is the probability that at least three workers take public transportation?

Exercise 8

Military radar and missile detection systems are designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a particular detection system has a .90 probability of detecting a missile attack. Use the binomial probability distribution to answer the following questions.

a. What is the probability that a single detection system will detect an attack?

b. If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack?

Exercise 8 cont'd

c. If three systems are installed, what is the probability that at least one of the systems will detect the attack?

d. Would you recommend that multiple detection systems be used? Explain.

Exercise 9

A university found that 20% of its students withdraw without completing the introductory statistics course. Assume that 10 students registered for the course.

a. Compute the probability that two or fewer will withdraw.

b. Compute the probability that at least 3 will withdraw.

c. Compute the expected number of withdrawals.

Exercise 10

Consider a Poisson distribution with $\mu = 3.$.

a. Write the appropriate Poisson probability function.

b. Compute $f(2)$.

c. Compute $P(X \geq 2)$.

Exercise 11

An average of 15 aircraft accident occur each year (*The World Almanac and Book of Facts, 2004*).

a. Compute the mean number of aircraft accident per month.
b. Compute the probability that there is no accidents during a month.
c. Compute the probability that there is exactly one accident during a month.
d. Compute the probability that there is more than one accident during a month.