BNAD/MGMT/ECON 276
Lecture 9
Statistical Inference III
Hypothesis Test

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Outline

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Introduction

- In the last lectures, we learned how to construct an interval that allows us to make a statement such as “We are 95% sure that the population mean μ is contained in the interval”.
- The point estimation and interval estimation still can not get the exact value of μ.
- In addition, we often make an hypothesis such as the population mean μ is equal to a number. (e.g. $\mu = 3$.)
- Then, we want to **test whether our hypothesis is correct or not.**
- In this lecture, we learn how to test such hypotheses using the sample data.

A Motivating Example

A company manufactures desks. The weekly production of desks follows a population distribution with a mean of 200 and a standard deviation of 16. Recently, this company has changed its production methods and new employees have been hired. The vice president of this company would like to investigate if there has been a change in the weekly production of desks.

- Before the change, the company figured out what is the mean and standard deviation of its weekly production with a long time statistical investigation.
- After the change occurred, it is reasonable to think that the weekly production of desks also changed.
- We want to test if the mean of weekly production has changed or not.

The Null Hypothesis and the Alternative Hypothesis

Null Hypothesis as an Assumption to be Challenged.

- We might begin with a belief or assumption that a statement about the value of a population parameter is true.
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect.
- In these situations, the null hypothesis is the assumption to be challenged

The Alternative Hypothesis is the negation of the Null Hypothesis.

Example 1, cont'd

- In the previous example, what we are interested in is whether there has been a change in the population mean $\mu = 200$.
- In this case, we can state the null hypothesis $H_0 : \ \mu = 200$, which means there has been no change in old mean 200.
- Then, the alternative hypothesis H_1 will be set up as follows. $H_1 : \ \mu \neq 200$, which means there has been change in the old mean 200.
- Keep in mind that, once H_0 has been set up, H_1 can be automatically set up as the "negation" of H_0.
- The alternative hypothesis can be denoted by either H_1 or H_a.

Example 2

The label on a soft drink bottle states that it contains at least 67.6 fluid ounces.

- The null hypothesis: $H_0 : \mu \geq 67.6$, i.e. the label is correct.
- The alternative hypothesis: $H_a : \mu < 67.6$, ie. the label is incorrect.

Example 3

A major west coast city provides one of the most comprehensive emergency medical services in the world. Operating in a multiple hospital system with approximately 20 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 12 minutes or less.
The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 12 minutes or less is being a achieved.

- State the testing hypotheses in this situation.

Summary of Forms for Null and Alternative Hypotheses about a Population Mean

- In general, the equality part of the hypothesis always appears in the null hypothesis.
- A hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean). There are three situations below:

$H_0 : \mu \geq \mu_0$	$H_0 : \mu \leq \mu_0$	$H_0 : \mu = \mu_0$
$H_a : \mu < \mu_0$	$H_a : \mu > \mu_0$	$H_a : \mu \neq \mu_0$
One-tailed test (lower tail)	one-tailed (upper tail)	two-tailed test

Reject and not rejecting H_0

- If we find a strong evidence against H_0, we say we reject H_0.
- If we find no evidence against H_0, we say we fail to reject H_0, or we accept H_0.
- The evidence comes from the data we have.

Type I and Type II Errors

- Since the evidence we will find comes from data and data can be changed depending on our sampling, our decision about H_0 will involve some mistakes.
- We can make two kinds of mistakes.
  - Type I error: reject H_0 when H_0 is True.
  - Type II error: accept H_0 when H_1 is True.
- The following table summarize these two errors.

	H_0 True	H_1 True
Accept H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct Decision

Level of Significance

- We can't avoid either Type I or Type II error.
- The usual way to handle this situation is as follows: (1) we commit Type I error to some degrees and (2) minimize Type II error.
- Usually, we fix Type I error to be either 0.1, 0.05, or 0.01.
- We need to know new terminology, the level of significance.

Level of Significance

The probability of making a Type I error.

- As said before, we will fix Type I error to be either 0.1, 0.05, or 0.01.
- Thus, our level of significance will be 0.1, 0.05, or 0.01 if we fix our Type I error to either 0.1, 0.05, or 0.01.

Outline

Hypothesis Test Step-by-Step Guideline

- Remind that our main interests is the hypothesis test about the relation between the population mean μ and the hypothesized population mean value μ_0.
- There are three steps when testing a hypothesis:

  1. Form the testing hypotheses.
  2. Calculate the test statistic. This step depends on two situations: σ known or σ unknown. For the former case, the test statistic is a z value. For the latter case, the test statistic is a t value.
  3. Use the rejection rule to draw conclusion. In this step, there are two methods. Either we use the p-value approach or we use the critical value approach.

- We have already learned how to do the first step.

2nd step: Calculate Test Statistic

Test statistic for the case σ known

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}}$$

If the H_0 is correct, then the z value above follows the standard normal distribution.

Test statistic for the case σ unknown

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{N}}$$

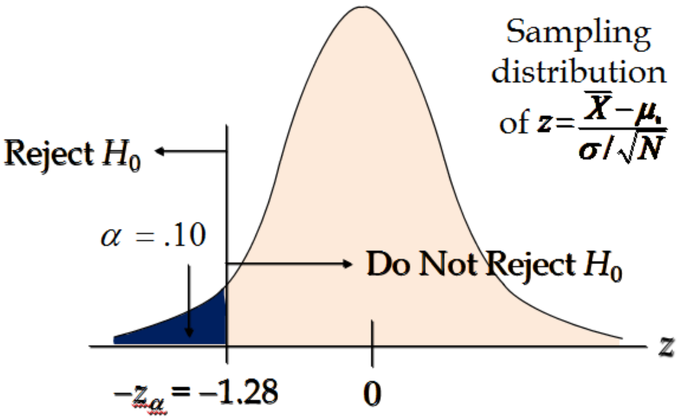
If the H_0 is correct, then the t value above follows the t distribution.

3rd Step: Use the Rejection Rule to Draw Conclusion

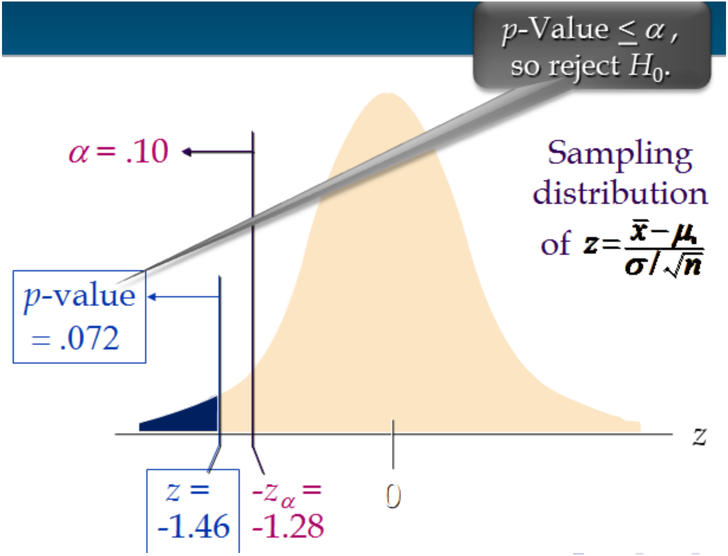
- There are two methods/approaches to draw conclusion.
- Both will give you the same conclusion.
- One approach is called the p-value approach. The other is the critical value approach.
- No matter what approach you use, you have to **use the z-table if σ is known** (so, you have calculated test statistic z), and **t-table if σ is unknown** (so, you have calculated test statistic t)
- The rejection rule in each approach varies with the form of the test hypothesis you have done in step 1.
- To make things simpler, we firstly learn in detail how to do step 3 with the specific case σ known (so, we use z-table).

Outline

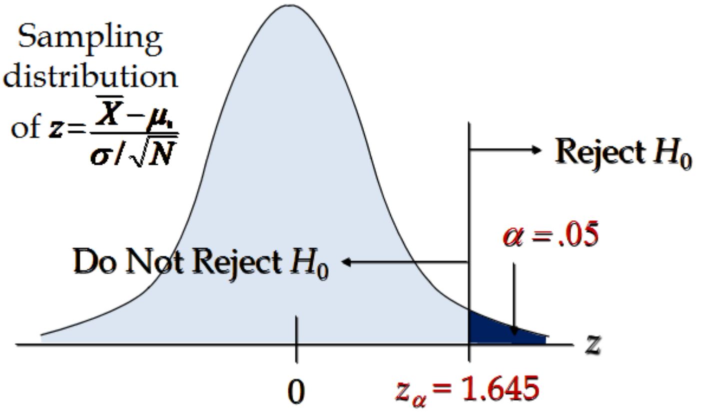
Critical Value Approach. Lower-Tail: Reject H_0 if $z \leq -z_\alpha$



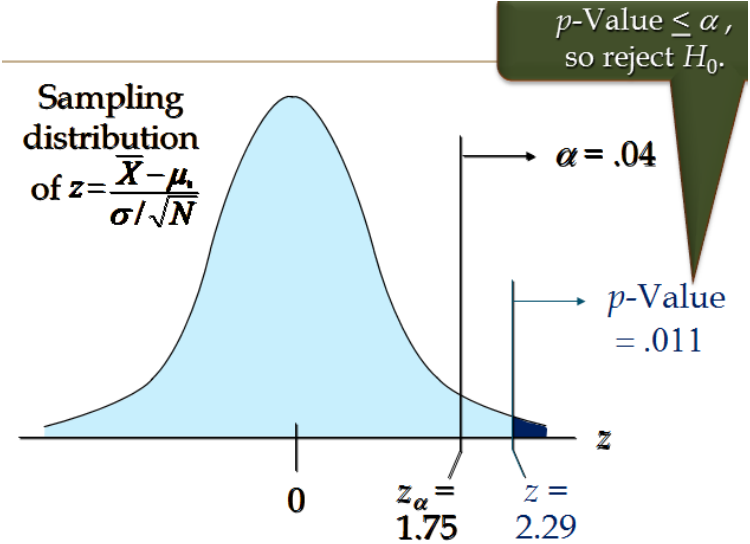
p-value Approach. Lower-Tail: Reject H_0 if p-value $\leq \alpha$



Critical Value Approach. Upper-Tail: Reject H_0 if $z \geq z_\alpha$



p-value Approach. Upper-Tail: Reject H_0 if p-value $\geq \alpha$



Example: Metro EMS

The response times for a random sample of 40 medical emergencies were tabulated. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.

The EMS director wants to perform a hypothesis test, with a 5% level of significance, to determine whether the service goal of 12 minutes or less is being achieved.

Example: Metro EMS, cont'd

1. Develop the hypotheses:

$$H_0 : \mu \leq 12$$
$$H_a : \mu > 12$$

2. Calculate the value of the test statistic:

$$z = \frac{\bar{X} - \mu_0}{\sigma / sqrtN} = \frac{13.25 - 12}{3.2/\sqrt{40}} = 2.47$$

3. Notice the level of significance is $\alpha = 0.05$ and the test here is lower tail test. There are two approaches to use.

Example, cont'd. Critical Value Approach

- Determine the critical value and rejection rule. For $\alpha = .05, z_{.05} = 1.645$
  Hence, the rejection rule is: Reject H_0 if $z \geq 1.645$
- Determine whether to reject H_0.
  Because $2.47 \geq 1.645$, we reject H_0. There is sufficient statistical evidence to infer that Metro EMS is **not** meeting the response goal of 12 minutes.

Example, cont'd. p-value Approach

- Calculate the p-value
  For $z = 2.47$, cumulative probability $= .9932$
  Hence, the p-value $= 1 - .9932 = .0068$
- Determine whether to reject H_0.
  Because p-value $= .0068 \leq \alpha = .05$, we reject H_0. There is sufficient statistical evidence to infer that Metro EMS is **not** meeting the response goal of 12 minutes.

Two-tailed Test. Critical Value Approach

1. Calculate the critical value corresponding to the new significant level $\alpha/2$. So, we get the critical value $z_{\alpha/2}$
2. Reject H_0 if $|z| \geq z_{\alpha/2}$

Two-tailed Test. p-value Approach

We compute the p-value using the following three steps:

1. Compute the value of the test statistic. Assume we are in the case σ known, so we use the test statistic z.
2. If z is in the upper tail ($z > 0$, compute the probability that z is greater than or equal to the value of the test statistic. For example, using the z-table to calculate $P(Z \geq z)$.
   If z is in the lower tail ($z < 0$), compute the probability the z is less than or equal to the value of the test statistic. For example, using the z-table to calculate $P(Z \leq z)$.
3. Double the tail area obtained in step 2 to obtain the p-value.

The rejection rule is: Reject H_0 if the p-value $\leq \alpha$.

Example: Glow Toothpaste

The production line for Glow toothpaste is designed to fill tubes with a mean weight of 6 oz. Periodically, a sample of 30 tubes will be selected in order to check the filling process.

Quality assurance procedures call for the continuation of the filling process if the sample results are consistent with the assumption that the mean filling weight for the population of toothpaste tubes is 6 oz; otherwise, the process will be adjusted.

Assume that a sample of 30 toothpaste tubes provides a sample mean of 6.1 oz. The population standard deviation is believed to be 0.2 oz. Perform a hypothesis test, at the 3% level of significance, to help determine whether the filling process should continue operating or be stopped and corrected.

Example, Glow Toothpaste, cont'd.

1. Determine the hypotheses:

$$H_0 : \mu = 6$$
$$H_a : \mu \neq 6$$

2. Calculate the test statistic. Here, since σ is known, the test statistic is z and then we will use z-table in step 3.

$$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} = \frac{6.1 - 6}{0.2/\sqrt{30}} = 2.74$$

3. Form the rejection rule and draw the conclusion by using either critical value approach or p-value approach.

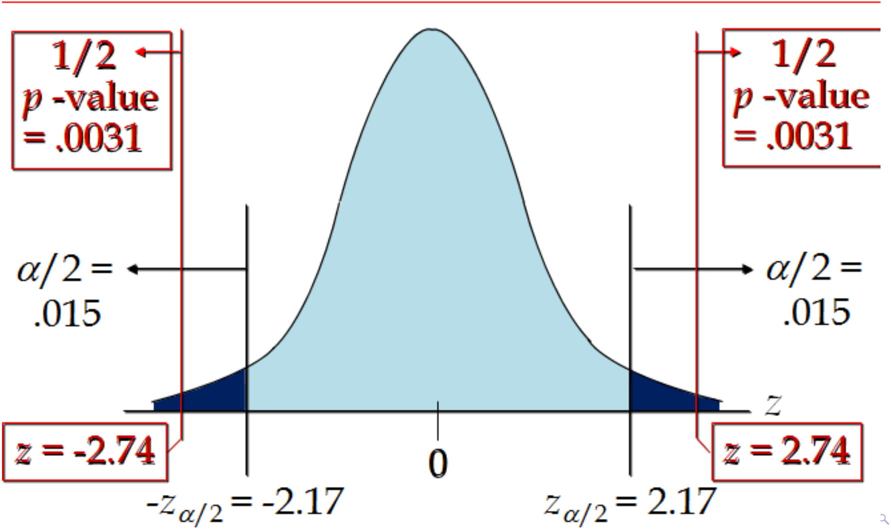
Example, cont'd. Critical Value Approach

1. Determine the critical value and rejection rule:
   For $\alpha/2 = .03/2 = .015$, $z_{0.015} = 2.17$
   The rejection rule is: Reject H_0 if $|z| \geq 2.17$.
2. Determine whether to reject H_0: Because $2.74 \geq 2.17$, we reject H_0.
   There is sufficient statistical evidence to infer that the alternative hypothesis is true. (i.e. the mean filling weight is not 6 ounces).

Example, cont'd, p-value Approach

1. Compute the p-value:
   For $z = 2.74$, cumulative probability $= .9969$. Hence,
   $P(Z \geq 2.74) = 1 - .9969$
   p-value $= 2(1 - .9969) = .0062$
2. Determine whether to reject H_0:
   Because p-value $= .0062 \leq \alpha = .03$, we reject H_0.
   There is sufficient statistical evidence to infer that the alternative hypothesis is true. (i.e. the mean filling weight is not 6 ounces).

Example of Two-Tailed Test, cont'd



Outline

Testing with σ unknown

There are two differences, compared to the case σ known.

- Instead of using test statistic z, we use test statistic t in which σ is replaced by the estimator s. (refer to slide 16)

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{N}}$$

- Instead of using the standard normal sampling distribution and Z-table, we will use the t-distribution and t-table for the step 3. Hence, all the p-value and critical values will be computed by using t and t_α (for one-tailed test) or $t_{\alpha/2}$ (for two-tailed test).

Outline

Exercise 1

Consider the following hypothesis test:

$$\begin{aligned} H_0 : \mu &= 22 \\ H_1 : \mu &\neq 22 \end{aligned}$$

A sample of 60 is used and the population standard deviation is 8. The sample mean is 51. For the following questions, use $\alpha = 0.05$.

a. Conduct Hypothesis Test with critical value approach and draw your conclusion.
   - a-1. What is the value of test statistics.
   - a-2. What is the critical values and critical region?
   - a-3. State your conclusion.

Exercise 1 cont'd

b. Conduct hypothesis test with p-value approach and draw your conclusion.

   b-1. What is p-value in this test?

   b-2. What is your conclusion? How did you use to draw your conclusion?

Exercise 2

Consider the following hypothesis test:

$$H_0 : \mu \leq 22$$
$$H_1 : \mu > 22$$

A sample of 25 provided a sample mean of 14 and a sample standard deviation of 4.32. Conduct Hypothesis Test with the significant value 5%.

2.1. What is the value of test statistics.

2.2. Test the hypothesis using the p-value approach and draw your conclusion.

2.3. Test the hypothesis using the critical value approach. State your conclusion.

Exercise 3

The average annual total return for U.S. Diversified Equity mutual funds from 1999 to 2003 was 4.1%. A researcher would like to conduct a hypothesis test to see whether the returns for mid-cap growth funds over the same period are significantly different from the average for U.S. Diversified Equity Funds.

a. Formulate the hypotheses that can be used.

b. A sample of 40 mid-cap growth funds provides a mean return of $\bar{X} = 3.4$. Assume that the population standard deviation for mid-cap growth funds is known and it is $\sigma = 2\%$. With $\alpha = 0.05$, conduct the hypothesis test with critical value approach.

c. With the same sample in the above, conduct the hypothesis test with p-value approach. Use the same α as in part (b). Draw your conclusion. Is your conclusion from the conclusion in part (b)?