Lecture 9
Statistical Inference III
Hypothesis Test

BNAD/MGMT/ECON 276

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Exercise 1

Consider the following hypothesis test:

$$\begin{aligned} H_0 : \mu &= 22 \\ H_1 : \mu &\neq 22 \end{aligned}$$

A sample of 60 is used and the population standard deviation is 8. The sample mean is 51. For the following questions, use $\alpha = 0.05$.

a. Conduct Hypothesis Test with critical value approach and draw your conclusion.

a-1. What is the critical values and critical region?

- Critical Values: -1.96 and 1.96, Critical Region: Regions that under the standard normal density and left from -1.96 and right from 1.96

a-2. What is the value of test statistics.

- Approximately, 28.081

a-3. State your conclusion.

- *State the decision rule based on the answers on part a-1. And, draw your conclusion.* Since 28.081 falls into the critical region, we decide to reject the null hypothesis, $H_0 : \mu = 22$.

Exercise 1 cont'd

b. Conduct hypothesis test with p-value approach and draw your conclusion.

b-1. What is p-value in this test?

- p-value$= 2 \times P(Z < -28.081) \approx 0$. *You can not find this value from Z-table but as you can see in the table, once you have a value less than -3.4, the probability that Z is less than that value is almost 0.*

b-2. What is your conclusion? How did you use p-value to draw your conclusion?

- *We reject H_0. We compare p-value with $\alpha = 0.05$. Then, if p-value is less than α we reject H_0. Otherwise, we don't reject the H_0.*

Exercise 2

Consider the following hypothesis test:

$$\begin{aligned} H_0 : \mu &\leq 22 \\ H_1 : \mu &> 22 \end{aligned}$$

A sample of 25 provided a sample mean of 14 and a sample standard deviation of 4.32. Conduct Hypothesis Test with the significant value 5%.

2.1. What is the value of test statistics.
Final result:

$$t = \frac{25 - 22}{4.32/\sqrt{25}} = 3.472$$

Exercise 2, cont'd

2.2. Test the hypothesis using the p-value approach and draw your conclusion.
Summarized answer: use the t-table, the degree of freedom is 24. $\alpha = .05$

$$p\text{-value} = P(T \leq -t) = P(T \geq t) = P(T \geq 3.47) \approx .001$$

Since $.001 < .05 = \alpha$, we reject H_0.

2.3. Test the hypothesis using the critical value approach. State your conclusion.
Summarized answer: critical value (at α of $.05$ and degree of freedom of 24) $= 1.711$. Since $|t| = 3.472 > 1.711$, we reject H_0.

Exercise 3

The average annual total return for U.S. Diversified Equity mutual funds from 1999 to 2003 was 4.1%. A researcher would like to conduct a hypothesis test to see whether the returns for mid-cap growth funds over the same period are significantly different from the average for U.S. Diversified Equity Funds.

a. Formulate the hypotheses that can be used.
   Ans: $H_0 : \mu = .041 = 4.1\%$ vs. $H_a : \mu \neq .041$

b. A sample of 40 mid-cap growth funds provides a mean return of $\bar{X} = 3.4$. Assume that the population standard deviation for mid-cap growth funds is known and it is $\sigma = 2\%$. With $\alpha = 0.05$, conduct the hypothesis test with critical value approach.
   Ans: Use z test statistic and z table. $z = -2.214$. This is the two-tailed test, so the critical value is $z_{.025} = 1.96$. Reject H_0.

c. With the same sample in the above, conduct the hypothesis test with p-value approach. Use the same α as in part (b). Draw your conclusion.

I will solve this in the class.