Nonlinear Pricing, Biased Consumers, and Regulatory Policy

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May 13, 2017

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Abstract

Price schedules with increasing block tariffs have been used by policy makers to regulate important markets such as water and electricity. Regulators sometimes argue that increasing block tariffs promote equity, resource conservation, and revenue stability. However, such tariffs are usually not argued to promote efficiency. This paper re-examines regulated non-linear pricing in light of recent evidence regarding how consumers respond to complex price schedules. Ito (2013) shows that, when confronted with complex price schedules, electricity customers respond to changes in average price (AP) rather than to changes in marginal price (MP). This paper characterizes the optimal regulated non-linear pricing when consumers respond to changes in average price. Optimal pricing under AP response behavior is shown to be independent of the consumer type distribution. A key result is that, fixing consumer preferences and the type distribution, increasing per-unit prices may be optimal when consumers respond to AP, while decreasing per-unit prices are optimal when consumers respond to MP. These results suggest that the equity-efficiency trade-off associated with increasing block tariffs may be less severe than previously believed.

Keywords: regulated non-linear pricing, increasing block tariffs

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1 Introduction

Nonlinear price schedules are observed for many goods and services. Firms offer quantity discounts on auto rentals, hotel room stays, and a wide variety of consumer goods. Cell phone service providers have complex nonlinear prices based on menus of three-part tariffs. Seminal papers by Mirrlees (1971), Spence (1977), Mussa and Rosen (1978), and Maskin and Riley (1984) show how firms with market power may use nonlinear pricing to engage in second-degree price discrimination and extract profits in excess of what could be achieved via uniform pricing. Profit-maximizing nonlinear price schedules exhibit decreasing marginal prices (i.e. quantity discounts) for a wide variety of environments, with the highest consumer type paying a marginal price equal to the firm's marginal cost.

Regulated firms also employ nonlinear pricing. For example, many regulated public utilities charge increasing block tariffs, in which consumers are charged higher marginal prices for higher consumed quantities. From an economic efficiency standpoint, increasing marginal price (IMP) schedule for a regulated firm is a puzzle. Suppose that the firm operates with economies of scale and the regulator wishes to set prices so as to maximize welfare subject to a break-even constraint for the firm. Then second-best optimal prices require distortions away from marginal cost, but these distortions will typically involve smaller mark-ups of marginal price over marginal cost for higher quantities; that is, decreasing marginal prices (which also follows the results in the context of non-regulated nonlinear pricing by Mirrlees (1971) and Maskin and Riley (1984)). The IMPs, represented by increasing block tariffs, however, are charged by many regulated public utilities since they are often argued to promote distributional goals.[1] For example, when energy costs increase significantly in the 1970s and 1980s, many electric utilities raised marginal rates on high-consumption service tiers, while keeping marginal rates low on lower tiers to protect lower income consumers. This, hence, points to the presence of an equity-efficiency trade-off for regulators, whereby distributional goals for pricing are achieved only by sacrificing efficiency. Borenstein (2012) provides evidence regarding the efficiency cost of increasing block tariffs used by California electric utilities though he analyzes the distortion away from setting an alternative flat rate schedule.

In this paper, we offer a different perspective on the equity-efficiency trade-off for price regulation. This perspective is based on recent empirical evidence on biased consumer responses to complex nonlinear price schedules. Ito (2014) uses customer billing data to examine how household electricity customers in California respond to changes in increasing block tariffs. He provides several types of empirical tests to show that consumers respond to changes in the average price of electricity, rather than to changes in marginal

[1] See for example, Wichelns (2013), who also argues that increasing block tariffs promote energy conservation goals.

price. We formulate and analyze a nonlinear pricing model to show that biased consumer responses to price changes can have dramatic effects on second-best optimal prices. For reasonable configurations of consumer preferences and distributions of consumer types, the shape of the second-best optimal nonlinear price schedule may be *reversed* for average price response (APR) consumers compared to that for marginal price response (MPR) consumers. In particular, second-best prices for APR consumers may involve increasing marginal prices whereas second-best prices for MPR consumers involve decreasing marginal prices. This finding suggests that regulated increasing marginal prices may not imply an equity-efficiency trade-off, or that this trade-off may not be as significant as prior studies have suggested.

Our model is built on the standard literature of monopolist's nonlinear pricing in Mussa and Rosen (1978), Maskin and Riley (1984), and Stole (2007). We extend to also consider regulated nonlinear pricing in which the regulator designs the price that maximizes social welfare subject to a break-even constraint on the monopolist's profits. However, we depart from standard literature by assuming consumers respond to average price instead of marginal price. This price misperception is due to the misspecified beliefs of the nonlinear structures of prices, subsidies, and tax schedules.[2] Hence, consumers who face a possibly nonlinear pricing menu act as if they faced a linear price in which marginal rates were also average rates.

Assuming APR, we find that optimal price schedule does not depend on the consumer's type distribution. Moreover, given a fixed preference, the optimal price schedule may exhibit increasing marginal prices under APR whereas it is decreasing marginal prices under MPR. This implies that increasing marginal price schedules in the world of biased consumers with APR may achieve double goals of efficiency and equity.

We also examine the welfare and the welfare distribution by the type of consumers between pricing in the world of APR and pricing in the world of MPR. Compared to the society with optimally pricing in the world of MPR, the regulated pricing solution in the world of APR may lead to a society with a higher welfare owing to a higher consumer surplus. That is because pricing in the world of APR may result in a lower marginal price schedule than pricing in the world of MPR. There may be a situation in which the welfare between the two worlds is the same. However, in such situation, pricing in the world of APR leads to a more favored distribution in consumer surplus. Particularly, low type consumers (who consume a low amount of goods) obtain more benefits in the world of APR pricing than in the world of MPR in two aspects. First, low type consumers who could not afford the goods in the world of MPR pricing are able to consume

[2]see evidences on Liebman and Zeckhauser (2004), Liebman (1998), and Fujii and Hawley (1988) on tax rates; Ito (2013), Borenstein (2009) on electricity price; and Carter and Milon (2005), and Saez (2010) on water price.

a positive amount in the world of APR pricing. Second, for low type consumers who can consume the goods in both worlds, low type consumers in the world of APR obtain higher surplus than in the world of MPR pricing. This is, again, because of a lower marginal price schedule in the world of APR, which reduces the distortion when pricing away the marginal cost. The deadweight loss caused by pricing away the marginal cost is mitigated to transfer into the consumer surplus.

There is a longstanding interest in studying misbehavior of consumers. In markets with nonlinear price schedules such as public utility pricing and taxation, Saez (2010) and Borenstein (2009) show that rational consumers respond to expected marginal price due to the presence of uncertainty about consumption. Using a general perceived price form that allows all three possible perceptions (marginal price, expected marginal price, average price), Ito (2014) provides strong empirical evidence that consumers respond to average price. Esponda and Pouzo (2016) develop a theoretical framework to explain the misspecified behaviors as called Berk-Nash equilibrium. They explain APR as a self Berk-Nash equilibrium in which a consumer has a misspecified subjective belief of the unit costs, leading her to act as if she faced a linear price though she faces a possibly nonlinear price schedule. We formulate the optimal regulated pricing when such price misperception in consumers is taken into account.

A growing literature of behavioral economics in industrial organization also examines how firms react to and in some cases exploit biased or suboptimal responses of consumers to complex pricing. Those studies suggest consumers have non-standard preferences (eg. loss aversion, or present bias), are overconfident or fail to choose the best price due to suboptimal search, etc (readers can refer to a comprehensive survey by DellaVigna (2009)). For example, Heidhues and Kőszegi (2008) and Spiegler (2012) suggest that loss aversion may create kinks in demand curves, which can lead to price rigidities. Courty and Hao (2000); Eliaz and Spiegler (2008); Grubb (2009) study monopoly screening problem or price discrimination when consumers are overconfident. Grubb (2009), especially, analyzes nonlinear pricing by a monopoly cell phone service provider. Consumers in his model are uncertain of their desired consumption level when they select a pricing plan and in addition exhibit a biased underestimate of the variance of desired consumption. The monopolist exploits this bias by offering a three-part tariff similar to commonly observed cell phone plans; a fixed fee, a set quantity of service with zero marginal price, and a high marginal price for overages above the set quantity. We consider the monopoly pricing and regulated pricing when consumers misperceive nonlinear prices as linear prices and hence respond to average prices instead of marginal prices. Our model help explain the optimality of increasing marginal prices.

The shape of nonlinear price schedules is also especially important in taxation literature.

The optimal shape of income tax rate has been well studied in Mirrlees (1971), Diamond (1998), and Dahan and Strawczynski (2000). Before the study by Diamond (1998), most analysis of optimal taxation supported decreasing marginal tax rates at high levels of income (see summary in Dahan and Strawczynski (2000)). Nevertheless, Diamond (1998) and the later comment by Dahan and Strawczynski (2000) demonstrated that the utility form and the worker skill distribution (equivalent to the consumer type distribution in product and service markets) account for the optimal tax rate shapes. The optimal tax rate, hence, may be increasing or U-shaped. We also consider the effect of utility form and type distribution on the shape of optimal marginal price but more importantly, the effect of APR rather than MPR. Liebman and Zeckhauser (2004) propose a "schmeduling" suboptimization in which consumers use average price as an approximation of marginal price due to the substantial cognitive cost of understanding complex pricing. Although Liebman and Zeckhauser (2004) model the scenario in which people respond to average tax rate, they do not study the impact on optimal tax shape. Furthermore, it is important to note that the nonlinear pricing model in labor economics, while similar to other product and service markets, exhibits significant differences in optimization constraints and in how uncertainty of the type of a consumer enters the preferences. Outside of labor economics (except Liebman and Zeckhauser (2004)), an exception that models the nonlinear pricing in APR is Sobel (1984). However, he only considers the conditions in which the monopolist optimal pricing schedule under APR has decreasing marginal prices. He does not fully contrast welfare between APR world and MPR world nor the distribution in welfare by the type of consumers.

The rest of the paper is organized as follows. Section 2 discusses the misperceived behaviors and the behavioral theory behind APR. Section 3 outlines the two pricing solutions—monopoly pricing and regulated firm pricing when consumers respond to marginal prices and when consumers respond to average prices. Section 4 presents the implications of APR pricing on the structure of the optimal price schedules. Section 5 shows the welfare implications. Section 6 concludes.

2 Modeling Average Price Response Behavior

In the monopoly pricing problem, the firm designs the price schedule (the pairs of price rate and consumption quantity) that maximizes expected profits given that consumers facing the price schedule will select the consumption to maximize individual utilities. That constraint is known as the incentive constraint where each type of consumers selects their own prescribed consumption and does not have incentive to buy the other consumptions (that are designed for other types).

In the world of MPR, a consumer of type θ selects a demand $q(\theta)$ in order to maximize their utility taking the entire total price schedule $P(q)$ as considered. Formally, it is written as

$$q(\theta) \text{ solves } \max \mathcal{U}(q, \theta, P(q)) \text{ for each } \theta \tag{1}$$

Under the conventional utility function $\mathcal{U}(q, \theta, P(q)) = U(q, \theta) - P(q)$, it is clear that the consumer of type θ compares the marginal utility to marginal price and opts to choose quantity $q(\theta)$ at which marginal utility equals marginal price, i.e $q(\theta)$ such that $U_q(q(\theta), \theta) = P_q(q(\theta))$ (subscripts denote derivatives).

In the world of APR, a consumer of type θ selects a demand $q(\theta)$ such that $U_q(q(\theta), \theta) = P(q(\theta))/q(\theta)$. That is because consumers misperceive the marginal price or the entire total price schedule because they have limited understanding of the price schedule or find the nonlinear price complicated to understand. We further elaborate the theory behind APR as follows.

Liebman and Zeckhauser (2004) identify conditions in three categories that can discourage people from perceiving the incentives at the margin. First, complexity, e.g. due to the nonlinear structure, makes it hard to determine marginal prices and costly to know where one is on the price schedule. Second, the connection between a consumer's choice and consumption is difficult to observe, especially in purchasing electricity and water. For example, how many kilowatts of electricity are used to cook a meal, and how much it costs to cook that meal is even more difficult to perceive. Third, nonstationary environment is not conducive for consumers to learning. Different seasons often lead consumers to stay at very different points on the price schedule (more heating is demanded in the winter). The pricing schedules are usually not displayed on the monthly bills and even the bills are presented in an incomprehensible measure such as kilowatts. The monthly payment also aggregates hundreds of disparate single activities (turning on the light, running the refrigerator, using the heater etc.) Therefore, Liebman and Zeckhauser propose that consumers perceive (or treat) the average price as the marginal price. This, which they call ironing, is because people smooth over the entire range of the schedule. "One decides whether to lower the heater by noting that \$300 per month represents an average price of 60 cents per therm, rather than 89 cents for the last (or next) therm of natural gas." (pg. 14).

In fact, Hortaçsu et al. (2015) imply a high possibility of APR instead of MPR behaviors. They document that the Public Utility Commission of Texas created a website that shows all available retail electricity provider options in order to inform consumers and provide transparency to the search process. That website, however, only displays average rates instead of marginal rates at different consumption plans. Such instruction may unintentionally lead consumers to focus on average prices rather than marginal prices.

Of course, people who are fiercely rational or who face very simple pricing schemes may know exactly their location on the price schedules and respond to marginal price. Some may use first-differencing to estimate marginal price or compare their own situation to that of a similar person to infer marginal price. Nevertheless, compared to marginal price, much less information is required to calculate average price (only the total payment and quantity are sufficient). Given the substantial cognitive cost of understanding complex pricing, consumers may respond to the average price of total payment as an approximation of their marginal price. This is empirically supported by Ito (2014). He shows that consumers do respond to price but they only perceive average price instead of marginal price because they are *inattentive* to the price schedule. In his model, he recovers the perceived price as $\tilde{p}(x) = \int p(x-\epsilon)w(\epsilon)d\epsilon$ where $p(x)$ is the marginal price of the price schedule, x is the consumption quantity, and $w(\epsilon)$ is uniformly distributed over $[0, x]$. In other words, consumers perceive the average price as the marginal prices weighted by uniformly distributed noise at smaller quantities. This is appropriate to the theory in Liebman and Zeckhauser (2004) proposing that consumers smooth over the entire range of the schedule and thus treat the average price as the marginal price.

All of the above papers, however, do not provide a formal theory to explain why consumers cannot learn about the marginal prices even if consumers may realize different average prices at different quantities. Sobel (1984) suggests a consumer who demands $q(\theta)$ thinks that she faces a linear price schedule with constant unit charge equal to $P(q(\theta))/q(\theta)$. In a formal model, it is written as:

$$q(\theta) \text{ solves } \max \mathcal{U}(q, \theta, P(q(\theta))/q(\theta)) \tag{2}$$

A reason for that, he explains, is that the consumer knows only the total amount she actually must pay for this purchase, not the entire price schedule. Sobel (1984) shows that consumers' behaviors adjust dynamically towards an APR convergence. The process suggests consumers respond to average prices because they know only the quantity and the total cost of their purchases. A consumer begins by making a purchase of q_0 and then learns about the value of $P(q_0)$. She makes the next demand under the assumption that the price is linear and the unit price is $P(q_0)/q_0$. In general, the consumer makes her nth purchase assuming the average price of the $(n-1)$th purchase is the constant unit cost. Sobel (1984) shows that this adjustment process converges to the problem (2). However, his theory of adjustment process is given under the conditions that lead to a decreasing marginal price schedule in the world of APR. The process cannot explain APR behavior in all types of price schedules.

A recent theory developed by Esponda and Pouzo (2016) can explain APR behaviors of consumers. They propose a concept of Berk-Nash equilibrium to model agents with misspecified environment. That is, instead of assuming people have a correctly specified view of their environment, each player is characterized by a subjective model which describes the set of feasible beliefs over payoff-relevant consequences as a function of actions. In a Berk-Nash equilibrium, each player follows a strategy that is optimal given her best fit belief. The notion of best fit belief is formalized in terms of minimizing the Kullback-Leibler divergence, which is endogenous and depends on the equilibrium strategy profile.

Applying their framework to the nonlinear pricing context, we see that the true environment is the set of all possible unit costs $\mathbb{Y} = \{y \in \mathbb{R} : y = p(q)/q, q \in \mathbb{R}\}$. The consumer incorrectly believes that she faces a (possibly random) linear price $y \in \mathbb{Y}$ that does not depend on her choice. Her misspecified subjective model is the set of all probability distributions over $\mathbb{Y}$. That is, $\Theta = \Delta(\mathbb{Y})$ where $\theta = (\theta_1, \ldots, \theta_k) \in \Theta$ and θ_j denotes the probability that the linear price is $P(q_j)/q_j$. The consumer's strategy is denoted by $\sigma = (\sigma_1, \ldots, \sigma_k) \in \Delta(\mathbb{Q})$, where σ_j is the probability that the consumer chooses quantity q_j. Esponda and Pouzo (2016) show that for a strategy σ, the weighted Kullback-Leibler divergence function is $K(\sigma, \theta) = \sum_{j=1}^{k} \sigma_j \ln \frac{1}{\theta_j}$ and the unique minimizer is $\theta(\sigma) = (\sigma_1, \ldots, \sigma_k)$. Given such best fit belief, the strategy σ also maximizes the consumer's surplus $\{U(q) - \left(\sum_{j=1}^{k}(P(q_j)/q_j)\sigma_j\right) q\}$. Thus, σ is a Berk-Nash equilibrium. A special case of σ that defines a pure strategy q_j and the belief of the linear price with unit cost $P(q_j)/q_j$ with probability of one is also a Berk-Nash equilibrium. This implies APR is a pure strategy Berk-Nash equilibrium.

Our model that characterizes nonlinear pricing in the world of APR fits with the APR behavior that is resulted from a pure strategy Berk-Nash equilibrium.

3 Defining Nonlinear Pricing Solutions

We consider the setting as in the standard model in Mirrlees (1971); Mussa and Rosen (1978) Maskin and Riley (1984); Wilson (1993). Given the total price schedule $P(q)$, a type-θ consumer whose preference is $U(q, \theta)$ has the surplus $U(q, \theta) - P(q)$ over the combination of quantity q and type θ; one-dimension type θ is distributed on $\Theta = [\theta_0, \theta_1]$ with density $f(\theta)$ and distribution $F(\theta)$. Assume that the outside option of not consuming is zero and firm faces a per-consumer cost $C(q)$ that is convex. Subscripts denote partial derivatives. Superscript m, a denotes the scenario where the consumer responds to marginal price and average price, respectively.

Assumption. Consumers' preferences satisfy:

1. $U_{qq} < 0$
2. $U_q > 0$
3. $U_\theta > 0$
4. $U_{q\theta} > 0$
5. $U_{q\theta} + qU_{qq\theta} > 0$
6. $2U_{qq} + qU_{qqq} < 0$

The first two assumptions ensure the positive marginal utility of consumption and the law of diminishing marginal utility. The third assumption aims to order the utility in the type of consumers. The fourth implies that the consumers with a higher type enjoy a higher marginal utility across every q (this is usually referred as the single-crossing condition). The last two assumptions guarantee monotonicity of consumption in consumer type under AP response price schedule.

It worths noticing that the consumer inverse demand in general is $U_q(q, \theta) = D_\theta(q) \equiv$ *Perceived Unit Price*. Under MP response, the consumer inverse demand is $U_q(q, \theta) = P_q(q) \equiv MP$. Under AP response, the consumer inverse demand is $U_q(q, \theta) = P(q)/q \equiv AP$.

We now define two pricing solutions, monopoly pricing and the regulated firm pricing (second-best pricing), for each case of price response behaviors.[3]

3.1 Monopoly Pricing Solution

3.1.1 Monopoly Pricing Under Marginal Price Response

As in the standard literature, the firm chooses the price schedule $P^m(q)$ to maximize expected profits subject to the incentive compatibility (IC) and individual rational (IR) constraints.

$$\max_{P^m(\cdot)} \int_{\theta_0}^{\theta_1} P^m(Q^m(\theta)) - C(Q^m(\theta))dF(\theta) \tag{3}$$

$$\text{subject to} \quad Q^m(\theta) = argmax_{q \geq 0} U(q, \theta) - P^m(q) \ [IC] \tag{4}$$

$$U(Q^m(\theta)) - P^m(Q^m(\theta)) \geq 0 \ [IR] \tag{5}$$

[3]Later, we can see that the two pricing solutions share a similar form of mark-up ratio formula and even the first best pricing can be included in that formula.

The standard results in the literature are that the optimal consumption $Q^m = \max\{q^m(\theta), 0\}$ where $q^m(\theta)$ is nondecreasing and satisfies (see appendix A for more detailed results):

$$U_q - C_q = \frac{1-F}{f} \cdot U_{q\theta} \tag{6}$$

It should be noticed that the optimal price schedule $P^m(q)$ has the marginal price equal to marginal utility and hence, we can rewrite equation 6:

$$\frac{MP - MC}{MP} = \frac{U_q - C_q}{U_q} = \frac{1-F}{f} \cdot \frac{U_{q\theta}}{U_q} \tag{7}$$

To interpret this formula, fix a quantity q and consider the demand for the qth unit of consumption. This unit has the price p^m, which is the *marginal price* since consumers are responding to marginal price. In other words, we have the marginal price demand in this case. The proportion of consumers willing to buy this unit is

$$D(p^m) \equiv 1 - F(\theta^*(p^m)) \tag{8}$$

$$\Rightarrow \frac{dD(p^m)}{dp^m} = f(\theta^*(p^m))\frac{d\theta}{dp^m} \tag{9}$$

where $\theta^*(p^m)$ denotes the type of consumer who is indifferent between buying and not buying the qth unit at marginal price p^m:

$$U_q(q, \theta(p^m)) = p^m \tag{10}$$

$$\Rightarrow U_{q\theta} d\theta = dp^m \tag{11}$$

$$\Rightarrow \frac{d\theta}{dp^m} = \frac{1}{U_{q\theta}} \tag{12}$$

Therefore,

$$\frac{dD(p^m)}{dp^m} \cdot \frac{p^m}{D(p^m)} = \frac{f(\theta^*(p^m))}{1 - F(\theta^*(p^m))} \cdot \frac{p^m}{U_{q\theta}} \tag{13}$$

Note that $p^m = U_q$, hence

$$\frac{MP - MC}{MP} = \frac{U_q - C_q}{U_q} = \frac{1}{\frac{f}{1-F}} \cdot \frac{U_{q\theta}}{U_q} = \frac{1}{\epsilon} \tag{14}$$

This means that the monopolist's mark-up ratio depends on the marginal price elasticity of demand of the qth unit.

3.1.2 Monopoly Pricing Under Average Price Response

As mentioned in section 2, under the APR case, the constraint (4) is replaced by the ex post behavioral incentive constraint:

$$U_q = \frac{P^a}{q} \tag{15}$$

The monopoly pricing solution is to choose the price schedule $P^a(q)$ to maximize expected profits:

$$\max_{P^a(\cdot)} \int_{\theta_0}^{\theta_1} P^a(Q^a(\theta)) - C(Q^a(\theta))dF(\theta) \tag{16}$$

$$\text{subject to} \quad Q^a(\theta) : U_q(Q^a(\theta)), \theta)) = \frac{P^a(Q(\theta))}{Q(\theta)} \tag{17}$$

$$U(q, \theta) - qU_q(q, \theta) \geq 0 \tag{18}$$

The profit maximization of the monopolist is transformed to

$$\max_{q} \int_{\theta_0}^{\theta_1} (qU_q(q, \theta) - C(q))dF(\theta) \tag{19}$$

$$\text{subject to} \quad U(q, \theta) - qU_q(q, \theta) \geq 0 \tag{20}$$

Since the constraint 20 is satisfied so long as the consumption is nonnegative, solving the problem requires maximizing the intergrand $qU_q(q, \theta) - C(q)$. This means that finding the monopolist's optimal AP response price schedule involves finding the most profitable uniform price for each θ-type consumer, and choosing the price schedule so that the average price for each type is equal to that corresponding most profitable uniform prices. The formal first order condition is at each θ:

$$U_q + qU_{qq} - C_q = 0 \tag{21}$$

$$U_q - C_q = -qU_{qq} \tag{22}$$

Apply the implicit function theorem to the above equation, we can see the change in optimal consumption in consumer type as follows:

$$\frac{dq}{d\theta} = -\frac{U_{q\theta} + qU_{qq\theta}}{2U_{qq} + qU_{qqq} - C_{qq}} \tag{23}$$

The assumptions (4) and (5) on the utility function and the weak convexity of the cost ensures the monotonicity and the uniqueness of the optimal consumption for each consumer type ($dq/d\theta > 0$). Since the monopolist shall designs the average price at the marginal utility of the optimal consumption, the price schedule is well defined in the sense that each consumer type facing such price schedule finds only one optimal consumption.

Lemma 1. *When the consumer responds to average price, the optimal consumption* $Q^a = \max\{q^a(\theta), 0\}$ *where* $q^a(\theta)$ *satisfies:*

$$U_q - C_q = -qU_{qq} \tag{24}$$

The optimal price schedule satisfies:

$$U_q = \frac{P^a}{q} \tag{25}$$

Equation 24 can be rewritten as:

$$\frac{U_q - C_q}{U_q} = \frac{-qU_{qq}}{U_q} \tag{26}$$

We now try to interpret the above mark up ratio by considering the *average price demand* in this case. Fix a type of consumer, the consumer of this type is different between buying and not buying the qth unit by comparing the marginal utility with the average price p^a of that unit:

$$U_q(q, \theta) = p^a \tag{27}$$

$$\Rightarrow U_{qq}dq = dp^a \tag{28}$$

$$\Rightarrow \frac{-qU_{qq}}{U_q} = -\frac{dp^a}{dq} \cdot \frac{q}{p^a} \equiv \frac{1}{\eta} \tag{29}$$

Therefore, we get

$$\frac{AP - MC}{AP} = \frac{U_q - C_q}{U_q} = \frac{-qU_{qq}}{U_q} = \frac{1}{\eta} \tag{30}$$

which means that the monopolist's mark-up price ratio depends on the *average* price elasticity of demand for each type of consumers since consumers respond to average price.

3.2 Regulated Firm Pricing (Second-best Pricing Solution)

Monopolist is often regulated by the price at which total welfare is maximized subject to the monopolist's profit at a fixed rate, often a break-even rate to just cover enough fixed costs F. This pricing method is also called Ramsey pricing which often refers to multiproduct contexts. We will consider the regulated monopoly pricing under MP response and under AP response. In fact, it later turns out that monopoly pricing and the regulated monopoly pricing, and even the first best solution, share a similar pricing form.

3.2.1 Regulated Firm Pricing Under Marginal Price Response

The regulator maximizes the social welfare subject to a constraint on the supplier's profit.

$$\max_{P^m(\cdot)} \int_{\theta_0}^{\theta_1} U(Q^m(\theta),\theta) - C(Q^m(\theta))dF(\theta) \tag{31}$$

$$\text{subject to} \quad \int_{\theta_0}^{\theta_1} P^m(Q^m(\theta)) - C(Q^m(\theta))dF(\theta) = FC \quad \text{[Profit constraint]} \tag{32}$$

$$Q^m(\theta) = argmax_{q\geq 0} U(q,\theta) - P^m(q) \quad \text{[IC constraint]} \tag{33}$$

$$U(Q^m(\theta)) - P^m(Q^m(\theta)) \geq 0 \quad \text{[IR constraint]} \tag{34}$$

Solving this problem (see Appendix B for more detail) yields that the optimal consumption $q^m = \max\{Q^m(\theta), 0\}$ where $q^m(\theta)$ satisfies:

$$U_q - C_q = \left(\frac{\lambda^m}{1+\lambda^m}\right) \cdot \left(\frac{1-F}{f} \cdot U_{q\theta}\right) \tag{35}$$

where λ^m is the Lagrangian multiplier for the monopolist's profit constraint. The Lagrangian term λ^m, hence, has the interpretation that it is the marginal increase in welfare associated with a decrease in firm profit.

Notice this means

$$\frac{MP - MC}{MP} = \frac{\mathcal{R}^m}{\epsilon} \tag{36}$$

where $\mathcal{R}^m \equiv \frac{\lambda^m}{1+\lambda^m}$ is a constant number and $\mathcal{R}^m \in (0,1)$ and ϵ is the marginal price elasticity of demand for qth unit across all types.

$$\epsilon = \frac{f}{1-F} \cdot \frac{U_q}{U_{q\theta}} \tag{37}$$

3.2.2 Regulated Firm Pricing Under Average Price Response

In contrast to the case where consumers respond to marginal price: $U_q = P^m_q$, the consumers here respond to average price (since they are not able to articulate the marginal price and use average as an estimate of marginal price to reason):

$$U_q = \frac{P^a}{q} \tag{38}$$

Therefore, the profit maximization of the monopolist is transfered to

$$\max_q \int_{\theta_0}^{\theta_1} (U(q,\theta) - C(q))dF(\theta) \tag{39}$$

$$\text{subject to} \quad \int_{\theta_0}^{\theta_1} (qU_q(q,\theta) - C(q))dF(\theta) = FC \tag{40}$$

$$U(q,\theta) - qU_q(q,\theta) \geq 0 \quad \text{[IR constraint]} \tag{41}$$

Since the IR constraint is satisfied so long as the consumption is nonnegative, we have the following FOC (with the Lagrangian multiplier λ^a for the profit constraint):

$$U_q - C_q + \lambda^a(U_q + qU_{qq} - C_q) = 0 \tag{42}$$

$$U_q - C_q = \frac{\lambda^a}{1+\lambda^a} \cdot (-qU_{qq}) \tag{43}$$

Theorem 1 (Regulated firm pricing under AP response)**.** *When the consumer responds to average price and under regulated pricing scheme, the optimal consumption* $q^a = \max\{Q^a(\theta), 0\}$ *where* $q = Q^a(\theta)$ *satisfies:*

$$U_q - C_q = \frac{\lambda^a}{1+\lambda^a} \cdot (-qU_{qq}) \tag{44}$$

The optimal price schedule satisfies:

$$U_q = \frac{P^a}{q} \tag{45}$$

□

This implies

$$\frac{U_q - C_q}{U_q} = \frac{\lambda^a}{1+\lambda^a} \cdot \frac{-qU_{qq}}{U_q} \tag{46}$$

$$\Leftrightarrow \frac{AP - MC}{AP} = \frac{\mathcal{R}^a}{\eta} \tag{47}$$

where η is the average price elasticity of demand for qth unit for each of the types of consumers. Similar to the MP response scenario, $\mathcal{R}^a$ is a constant between 0 and 1.

The optimal mark-up price ratios imply that the monopoly pricing and the regulated firm pricing share a similar form.

$$\begin{aligned} &\text{Under MP response} && \text{Under AP response} && (48)\\ &\frac{MP - MC}{MP} = \frac{\mathcal{R}^{(m)}}{\epsilon} && \frac{AP - MC}{AP} = \frac{\mathcal{R}^{(a)}}{\eta} && (49) \end{aligned}$$

It can be seen that the monopoly pricing, the first-best pricing, and the regulated monopoly pricing are encompassed by $\mathcal{R} = 1$, $\mathcal{R} = 0$, and $0 < \mathcal{R} < 1$, or by $\lambda \to \infty$, $\lambda = 0$, $0 < \lambda < \infty$, respectively[4]. In this paper, we call the constant $\mathcal{R}$ the policy number.

4 Pricing Structure Implications

In this section, we will consider the implications of the price response behaviors on the optimal price schedules. Recall that the two pricing solutions share the similar form (49), the proofs will use that general form although the intuition will be provided by working with the monopoly pricing for tractability.

Corollary 1. *When consumers respond to average price, the optimal pricing schedule does not depend on the consumer type distribution.*

Proof. Remind that the optimal pricing schedule under AP response satisfies the equations 15 and 24, which are independent of the type distribution. The intuition for this is that the monopolist can maximize total profits by maximizing profits point-wise across types. □

This appears to be surprising since the optimal mark-up pricing schedule under to-marginal-price response relies on the type distribution. However, figure 1 gives us the reason why. To simplify the argument, suppose the monopolist has zero cost and only needs to maximize the revenue. Consider any two types of consumers a, b, that have demand curves AB and CD, respectively. It should be also noticed that these are demand curve to which price the consumers respond. Hence, the intersection between the demand curves and the perceived price is the quantity that consumers purchase at a given price rate.

Consider the situation in which consumers respond to average price. If the monopolist designs the price rate d—average rate to which consumers respond, then the type a consumer responds at point F and type b responds at point G. The monopolist's total

[4] Similar result for Ramsey pricing in the case that consumers respond to marginal price is obtained in Braeutigam (1989), section 7.3.

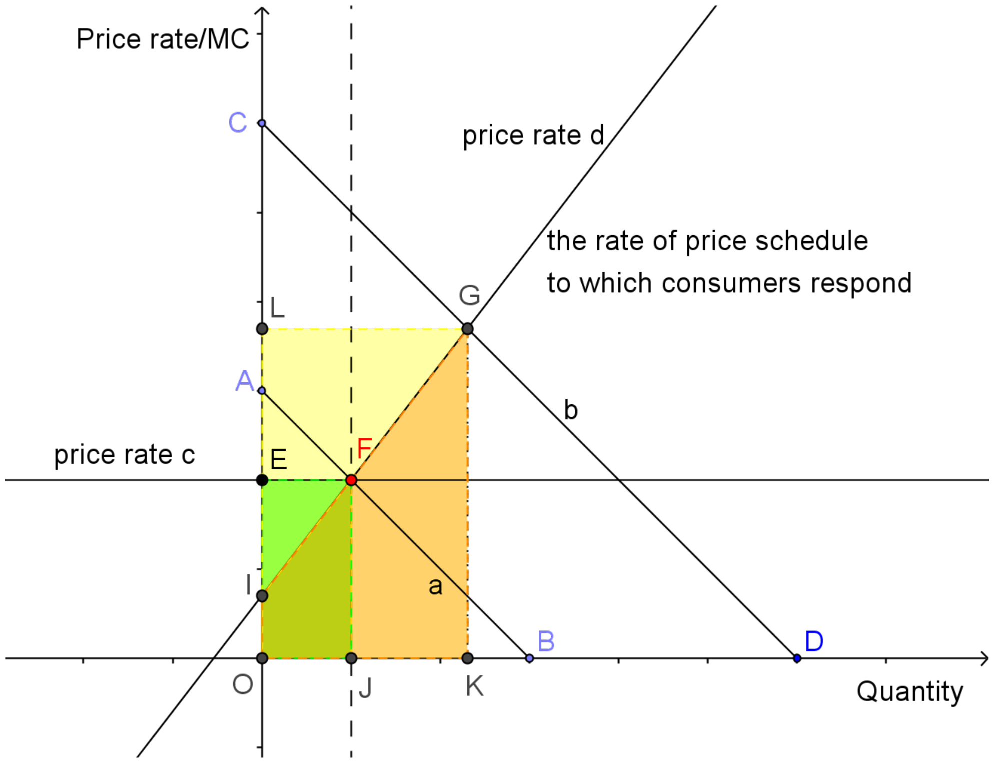


FIGURE 1: Monopolist's revenue under different response schemes.

Note: The horizontal axis is for quantity, the vertical axis is for the price rate to which consumers respond (either marginal rate or average rate, depending on how consumers respond to).

revenue consists the revenue $OEFJ$ collected in the type segment a and the revenue $OLGK$ collected in the segment b. We can see that the revenue in each segment does not depend on the price rate at other quantities except the quantity at which the consumers purchase. Particularly, the revenue $OLGK$ in type segment b is only determined by the specific design point G and not affected by other price rates designed at other quantities for other types.

On the contrary, if consumers respond to marginal price then the monopolist shall design the optimal marginal price scheme. If the marginal price rate is designed as the line d, the monopolist's revenue obtained from selling to consumer type a is $OIFJ$ and the revenue obtained in the type segment b is $OIGK$. We can see that the price design IG set on the first quantity unit to the final quantity unit (q^G) demanded by type b plays the role as a constraint on how much the monopolist can get from serving the segment b.

In summary, the pairs of price and quantity in the price schedule under AP response scheme are independent (disjoint relationship) thanks to two features. First, since consumers respond to average price, the monopolist can be rational by design the average price schedule. Second, the average price points are independent in the sense that the total revenue generated by any pair of price and quantity in the schedule does not depend on the other pairs.

The disjoint relationship among the monopolist's designed pairs of price and quantity when consumers respond to average price leads to an interesting result. As long as each consumer type demands different optimal quantities, the monopolist can just focus on maximizing each revenue obtained from selling to each type of consumers under AP response situation. Such action is in fact a point wise maximization in the sense that maximizing revenue obtained from selling to each type of consumers implies maximizing total revenue obtained from serving the whole market. As a result, the optimal price schedule does not depend on the distribution of consumer types[5].

Corollary 2 (Quantity Discount)**.** *When consumers respond to marginal price, the optimal pricing schedule has decreasing marginal rate if the marginal price elasticity of demand across all types of consumers is increasing in consuming amount.*

Corollary 3 (Quantity Premium)**.** *When consumers respond to average price, the optimal pricing schedule is increasing average rate if the average price elasticity of demand for each type of consumers is decreasing in consumption amount.*

It should notice that increasing marginal rate implies the increasing average rate and vice versa.[6] Then the two results here appear to exclude each other but in fact they can hold together. In particular, there are situations in which the marginal price elasticity is increasing but the average price elasticity is decreasing.

If we have the utility form $U(q, \theta) = \theta V(q)$ then the condition in proposition 2 is equivalent to that the hazard rate is increasing. This condition is well known in literature by Mussa and Rosen (1978) and Maskin and Riley (1984). Examples of increasing hazard rate include regular distributions such as uniform. Meanwhile, the decreasing hazard rate, for example, Pareto distribution would imply the decreasing marginal rate pricing schedule.

Corollary 4. *Assume the utility form $U(q, \theta) = \theta V(q)$. When the consumers respond to marginal price, the optimal pricing schedule is decreasing average rate if the hazard rate of the type distribution is increasing. Meanwhile, when the consumers respond to average price, the optimal pricing schedule is increasing average rate if the relative curvature $\frac{-qV''}{V'}$ is increasing.*

[5]Meanwhile, under to-marginal-price response situation, the price for a qth unit will affect the monopolist's revenue from selling to all types that demand quantities at least q units. For that reason, under the MP response situation, the monopolist has to take into account the type distribution to quantify the weighted demand for the qth unit.

[6]Suppose the price schedule P has the marginal price P_q and average price P/q. The rate change due to the change in q of the average price is $\frac{dP/q}{dq} = \frac{P_q \cdot q - P}{q^2}$. Consider the function $g(q) = qP_q - P$ that has $g_q = qP_{qq}$. Hence, decreasing marginal price means that $P_{qq} < 0$, which implies $g(q)$ is decreasing, i.e. $g(p) < g(0) = 0$. Hence, $P_q \cdot q - P < 0$. This means $\frac{dP/q}{dq} < 0$, i.e. decreasing average price.

5 Welfare

5.1 The Distributional Effects in Consumer Welfare In the World of APR Price Schedule

Remark. In both the monopoly pricing solution and regulated firm pricing solution, APR may lead to a distributional effects in consumer welfare. Particularly, while both monopolist's profits and total consumer surplus are the same between MPR price schedule and APR price schedule, the distribution of consumer welfare by consumer types is contrast. Low type consumers may benefit in the world of APR price schedule while suffer in the wolrd of MPR price schedule.

To illustrate this result, we use an example with a quadratic utility function $U(q,\theta) = (1+\theta)q - 1/2q^2$; $C(q) = cq$, and fixed cost F. Consumer type is uniformly distributed from 0 to 1, i.e. $\theta \sim Uniform[0,1]$. Assume $1 \leq c \leq 2$.

5.1.1 Marginal Price Response, with Uniform Distributed Type

The cutoff type above which consumers can consume positive amounts of goods is:

$$\theta \geq \frac{c + \mathcal{R}^m - 1}{\mathcal{R}^m + 1} \tag{50}$$

Optimal consumption is

$$Q^{m,Uniform}(\theta) = \begin{cases} (\mathcal{R}^m + 1)\theta - \mathcal{R}^m + 1 - c, & \theta \geq \frac{c+\mathcal{R}^m-1}{\mathcal{R}^m+1} \\ 0, & \theta < \frac{c+\mathcal{R}^m-1}{\mathcal{R}^m+1} \end{cases} \tag{51}$$

The optimal pricing schedule satisfies

$$P_q^{m,Uniform} = \frac{c + 2\mathcal{R}^m}{\mathcal{R}^m + 1} - \frac{\mathcal{R}^m}{\mathcal{R}^m + 1} \cdot q, \text{ for } \theta \geq \frac{c + \mathcal{R}^m - 1}{\mathcal{R}^m + 1} \tag{52}$$

Expected profits before the fixed cost:

$$\mathbf{E}\pi^{m,Uniform} = \frac{\mathcal{R}^m}{3(\mathcal{R}^m + 1)^2}(2 - c)^3 \tag{53}$$

Consumer surplus and expected consumer surplus:

$$CS^{m,Uniform} = \frac{((\mathcal{R}^m + 1)\theta - c - \mathcal{R}^m + 1)^2}{2(\mathcal{R}^m + 1)} \quad \text{for } \theta \geq \frac{c + \mathcal{R}^m - 1}{\mathcal{R}^m + 1} \tag{54}$$

$$\mathbf{E}[CS^m] = \frac{(2 - c)^3}{6(\mathcal{R}^m + 1)^2} \tag{55}$$

5.1.2 Average Price Response, with Uniform Distributed Type

The cutoff type above which consumers can consume positive amounts of goods is:

$$\theta \geq c - 1 \tag{56}$$

This cutoff type is higher than the cutoff type under MP response case since $\frac{c+\mathcal{R}^m-1}{\mathcal{R}^m+1} \geq c - 1 \ \forall \mathcal{R}^m \geq 0$.

The optimal consumption is

$$Q^{a,Uniform}(\theta) = \begin{cases} \frac{\theta+1-c}{\mathcal{R}^a+1}, & \theta \geq c - 1 \\ 0, & \theta < c - 1 \end{cases} \tag{57}$$

The optimal pricing schedule satisfies

$$AP^{a,Uniform} = \mathcal{R}^a q + c, \text{ for } \ \theta \geq c - 1 \tag{58}$$

Expected profits before the fixed cost:

$$\mathbf{E}\pi^{a,Uniform} = \frac{\mathcal{R}^a}{3(\mathcal{R}^a + 1)^2}(2 - c)^3 \tag{59}$$

Consumer surplus and expected consumer surplus:

$$CS^a = \frac{(\theta - c + 1)^2}{2(\mathcal{R}^a + 1)^2} \quad \text{for } \theta \geq c - 1 \tag{60}$$

$$\mathbf{E}[CS^a] = \frac{(2 - c)^3}{6(\mathcal{R}^a + 1)^2} \tag{61}$$

We see that in both the case of monopoly pricing ($\mathcal{R}^m = \mathcal{R}^a = 1$) and the case of regulated firm pricing, the expected before-the-fixed-cost profits remain unchanged:

$$\mathbf{E}\pi^{m,Uniform} = \mathbf{E}\pi^{a,Uniform} \tag{62}$$

The same as the expected consumer surplus, and hence social welfare (since $\mathcal{R}^m = \mathcal{R}^a$ due to the constant profit constraint in the regulated firm pricing and $\mathcal{R}^m = \mathcal{R}^a = 1$ in the monopoly pricing):

$$\mathbf{E}[CS^{a,Uniform}] = \mathbf{E}[CS^{m,Uniform}] \tag{63}$$

However, the price response behaviors lead to a surplus distribution that favors equity view. Notice the change in the difference in consumer surplus between APR and MPR as consumer type changes:

For those types who can buy the goods, i.e. $\theta \geq c/2$:

$$\begin{aligned} CS^{a,Uniform} - CS^{m,Uniform} = -\frac{\mathcal{R}}{2(\mathcal{R}+1)^2}\big[(\mathcal{R}^2+3\mathcal{R}+3)\theta^2 - 2(\mathcal{R}^2+\mathcal{R}+\mathcal{R}c+2c-1)\theta \\ +\mathcal{R}^2+2\mathcal{R}c+c^2-\mathcal{R}-1\big] \end{aligned} \tag{64}$$

While for those consumer types who are not able to buy the goods under MPR price schedule, i.e. for all $\theta \in [c-1; c/2)$: $CS^{a,Uniform} > 0 = CS^{m,Uniform}$

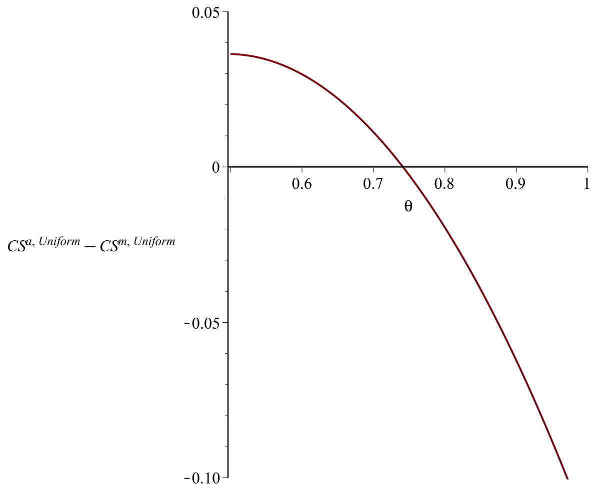


FIGURE 2: The difference in consumer surplus between the APR scheme and the MPR scheme in the case of uniform distribution.

Note: The horizontal axis is for the type θ, the vertical axis is for the difference in consumer surplus $CS^{m,Uniform} - CS^{a,Uniform}$. Parameter values are $c = 1, \mathcal{R} = 0.6$.
Notice $CS^{a,Uniform} > 0 = CS^{m,Uniform}$, for $\theta \in (0, 0.5)$. For $\theta \in (0.5, 1)$, the difference in consumer surplus is depicted as in this graph.

Figure 2 graphs an example of the difference in consumer surplus between the APR scheme and the MPR scheme based on the relation (64). We can see that low type benefits from APR in two points. First, there are some low types can consume the goods under APR but not under MPR (the type θ between $c-1$ and $c/2$). Second, among the consumers that buy the goods ($\theta \geq c/2$), the low types get higher surplus under APR than MPR while the high types get smaller surplus under APR. Therefore, even though the expected social welfare under the two price response cases are the same, the high types in fact subsidize the low types under APR case.

That story can also be captured in figure 3. Figure 3 shows the optimal price schedules

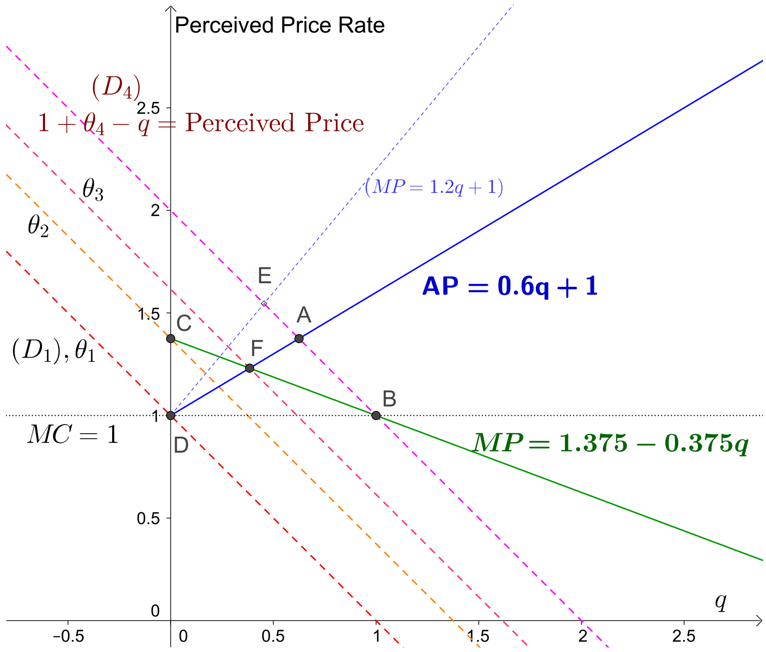


FIGURE 3: The regulated price design under APR and MPR in the case of uniform distribution and linear demands

Note: There are two optimal price schedules corresponding to two price response behaviors. When consumers respond to marginal price, the regulator opts to design $MP = 1.375 - 0.375q$. When consumers respond to average price, the regulator opts to design $AP = 0.6q + 1$. Under the quadratic utility function and uniform distribution of consumer type, the APR price schedule is IUP while the MPR price schedule is DUP.

under APR and MPR in the case of $\mathcal{R} = 0.6$ and $c = 1$. We can see that those consumer types between θ_1 to θ_2 could not pay for the goods under MPR price schedule though they can under APR price schedule. That is because the price rate under MPR price schedule is much higher than the price rate under APR price schedule. Such price rate gap remains until the intersection quantity point F in the graph. This, therefore, implies that among consumers who can pay for the goods under both price response cases, there still exists the low types – between θ_2 and θ_3 – who enjoy larger surplus under APR price schedule than under MPR price schedule (while the high types from θ_3 to θ_4 experience a contrary situation).

5.2 Suboptimal Behavior May Increase Social Welfare Compared To Standard Optimal Behavior

Remark. In the monopoly pricing solution, the firm may get better off and consumers are worse off for the APR of the consumers.

In the regulated firm pricing solution, APR price schedule may lead to an increase in welfare compared to MPR pricing.

To formally verify the above remark, we will work with a quadratic utility function that leads to linear demands, and the Pareto distribution of the consumer type. Specifically, let $U(q,\theta) = (1+\theta)q - 1/2q^2$; $C(q) = cq$, and a fixed cost F. The consumer type follows Pareto distribution with scale $s > 0$ and shape $\alpha > 0$ that has the following probability density function (PDF) and the cumulative distribution function (CDF) respectively:

$$f(x) = \frac{\alpha s^\alpha}{x^{\alpha+1}} \tag{65}$$

$$F(x) = 1 - \frac{s^\alpha}{x^\alpha} \tag{66}$$

The hazard rate $\frac{f(\theta)}{1-F(\theta)} = \frac{\alpha}{\theta}$. See figure 4 for the density shape of Pareto distribution under different values of parameters. An important note is the support of Pareto distribution is (s,∞). If we think the type is represented by income, Pareto distribution is appropriate to depict an economy where everyone has a definitely positive income amount and there is no restriction on the highest income amount someone can earn.

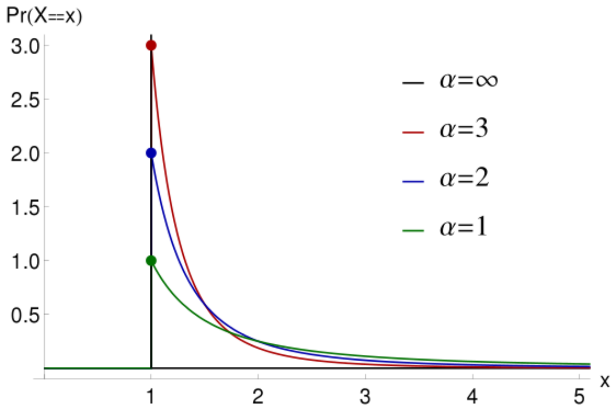


FIGURE 4: This figure depicts the Pareto density with scale $s = 1$ and different shape κ.

We first present the main results in MPR case and APR case then discuss the results under monopoly pricing and regulated firm pricing. Recall that the monopoly pricing and regulated firm pricing share the similar pricing form using the policy number $\mathcal{R}$ in the mark-up ratio. The monopoly pricing corresponds to the case $\mathcal{R} = 1$ and the regulated firm pricing corresponds to the case $\mathcal{R} \in (0,1)$. We have the following results:

5.2.1 Marginal Price Response With Pareto Distributed Type

The cutoff type above which consumers can consume a positive amount of goods is:

$$\theta \geq \frac{c-1}{1-\mathcal{R}^m/\alpha} \tag{67}$$

The optimal consumption is

$$Q^{m,Pareto}(\theta) = \begin{cases} \left(1-\frac{\mathcal{R}^m}{\alpha}\right)\theta + 1 - c, & \theta \geq \frac{c-1}{1-\mathcal{R}^m/\alpha} \\ 0, & \text{otherwise} \end{cases} \tag{68}$$

The optimal pricing schedule satisfies

$$MP^{m,Pareto} = P_q^{m,Pareto} = \left(\frac{1}{1-\frac{\mathcal{R}^m}{\alpha}} - 1\right) q + \frac{c-1}{1-\frac{\mathcal{R}^m}{\alpha}} + 1 \tag{69}$$

Expected before-the-fixed-cost profits:

$$\mathbf{E}\pi^{m,Pareto} = \frac{\left(1-\frac{\mathcal{R}^m}{\alpha}\right)^{\alpha} \mathcal{R}^m (c-1)^{2-\alpha} s^{\alpha}}{(\alpha-\mathcal{R}^m)(\alpha-2)}, \text{ for } \alpha > 2 \text{ and } c-1 \geq s \tag{70}$$

Expected consumer surplus:

$$\mathbf{E}[CS^{m,Pareto}] = \frac{(c-1)^{2-\alpha}\alpha^2 s^{\alpha}\left(1-\frac{\mathcal{R}^m}{\alpha}\right)^{\alpha+1}}{(\alpha-\mathcal{R}^m)^2(\alpha-1)(\alpha-2)}, \text{ for } \alpha > 2 \text{ and } c-1 \geq s \tag{71}$$

5.2.2 Average Price Response with Pareto Distributed Type

The cutoff type above which consumers can consume positive amounts of goods is:

$$\theta \geq c-1 \tag{72}$$

The optimal consumption is

$$Q^{a,Pareto}(\theta) = \begin{cases} \frac{\theta+1-c}{1+\mathcal{R}^a}, & \theta \geq c-1 \\ 0, & \theta < c-1 \end{cases} \tag{73}$$

The optimal pricing schedule satisfies

$$AP^{a,Pareto} = \mathcal{R}^a q + c, \ \theta \geq c-1 \tag{74}$$

Expected before-the-fixed-cost profits:

$$\mathbf{E}\pi^{a,Pareto} = \frac{2\mathcal{R}^a(c-1)^{2-\alpha}s^\alpha}{(\mathcal{R}^a+1)^2(\alpha-2)(\alpha-1)}, \text{ for } \alpha > 2 \text{ and } c-1 \geq s \tag{75}$$

Expected consumer surplus:

$$\mathbf{E}[CS^{a,Pareto}] = \frac{s^\alpha(c-1)^{2-\alpha}}{(\mathcal{R}^a+1)^2(\alpha-2)(\alpha-1)},, \text{ for } \alpha > 2 \text{ and } c-1 \geq s \tag{76}$$

5.2.3 Welfare Implication Under Monopoly Pricing

Under monopoly pricing solution ($\mathcal{R}^m = \mathcal{R}^a = 1$), we see that

$$\mathbf{E}\pi^{m,Pareto} < \mathbf{E}\pi^{a,Pareto} \tag{77}$$

This means that the monopolist gets better off due to the inattention to the price of consumers. When consumers respond to average price, they would face a higher increasing slope of the average rate price schedule than under MPR scenario. The steeper increasing unit rate price schedule would bring higher before-the-fixed-cost profits to the monopolist. This also implies that consumers in general would get worse off for their biased behaviors and indeed, we have:

$$\mathbf{E}[CS^{a,Pareto}] - \mathbf{E}[CS^{m,Pareto}] = -\frac{\left(1-\frac{1}{4}\left(1-\frac{1}{\alpha}\right)^{1-\alpha}\right)\alpha^2 s^\alpha (c-1)^3}{(\alpha-2)(\alpha-1)^3(c-1)^{\alpha+1}\left(1-\frac{1}{\alpha}\right)^{1-\alpha}} < 0 \tag{78}$$

5.2.4 Welfare Implication Under Regulated Firm Pricing

We are interested in the existence of the situation in which the regulated firm can covers fixed costs to break even while the social welfare is higher under APR than MPR. We will show that such situation exists under appropriate policy parameters $\mathcal{R}^a, \mathcal{R}^m$ and distribution parameter α.

The monopolist's before-the-fixed-cost profits are regulated to a fixed return to cover just enough the same fixed costs under both APR and MPR. That is, the constant Ramsey numbers $\mathcal{R}^m$ and $\mathcal{R}^a$ satisfy:

$$\frac{\left(1-\frac{\mathcal{R}^m}{\alpha}\right)^{\alpha}\mathcal{R}^m(c-1)^{2-\alpha}s^{\alpha}}{(\alpha-\mathcal{R}^m)(\alpha-2)} = F = \frac{2\mathcal{R}^a(c-1)^{2-\alpha}s^{\alpha}}{(\mathcal{R}^a+1)^2(\alpha-2)(\alpha-1)}, \text{ for } \alpha > 2 \text{ and } c-1 \geq s \tag{79}$$

$$\Rightarrow \left(1-\frac{\mathcal{R}^m}{\alpha}\right)^{\alpha-1}\frac{\mathcal{R}^m}{\alpha} = \frac{2\mathcal{R}^a}{(\mathcal{R}^a+1)^2}(\alpha-1) \tag{80}$$

Notice the difference in expected consumer surplus (same as in expected social welfare since the expected variable profits equal to the fixed cost F) between the two price response cases is:

$$\mathbf{E}[CS^{a,Pareto}] - \mathbf{E}[CS^{m,Pareto}] = \frac{s^{\alpha}(c-1)^{2-\alpha}}{(\alpha-1)(\alpha-2)}\left((\mathcal{R}^a+1)^{-2} - \left(1-\frac{\mathcal{R}^m}{\alpha}\right)^{\alpha-1}\right) \tag{81}$$

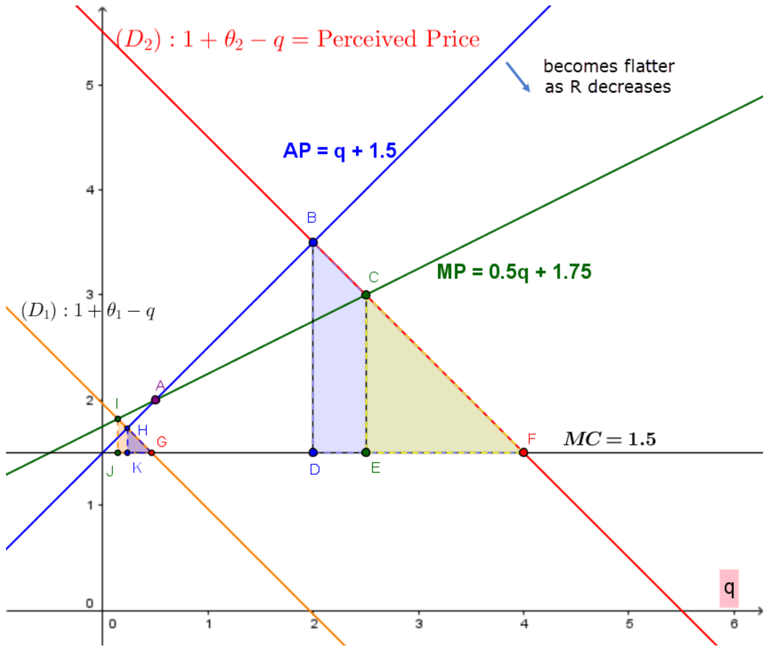
Using the equation 80, we have

$$\mathbf{E}[CS^{a,Pareto}] - \mathbf{E}[CS^{m,Pareto}] = \frac{s^{\alpha}(c-1)^{2-\alpha}}{(\alpha-1)(\alpha-2)}(\mathcal{R}^a+1)^{-2}\left(1-2\cdot\frac{\mathcal{R}^a}{\mathcal{R}^m}\cdot\frac{\alpha}{\alpha-1}\right) \tag{82}$$

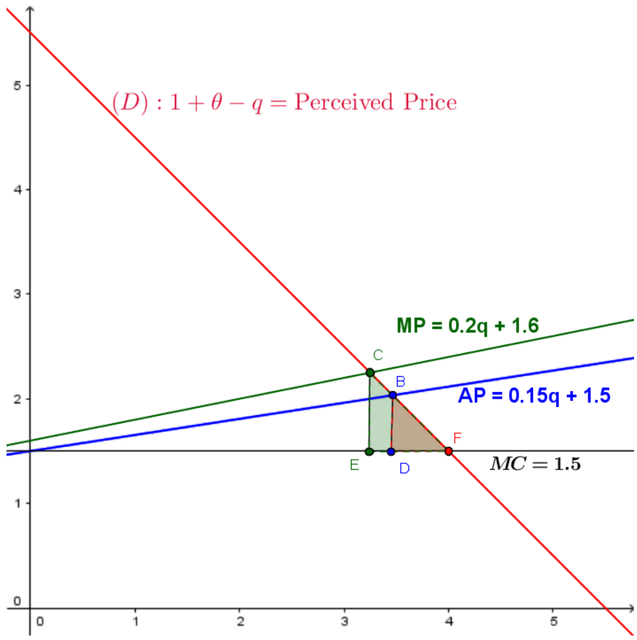
Hence, the regulated monopoly pricing under APR will lead to higher social welfare than under MPR as long as the policy allows parameters $\mathcal{R}^m, \mathcal{R}^a$ appropriately to demand distribution environment α such that the relation (80) holds and $\frac{\mathcal{R}^a}{\mathcal{R}^m} < \frac{\alpha-1}{2\alpha}$. An example is the policy that induces $\mathcal{R}^m = 0.5$ when $\alpha = 3$, which leads to $\mathcal{R}^a = 0.154$.

Figure 5 shows the change in optimal price schedules when changing from monopoly pricing to regulated firm pricing and from MPR to APR. In monopoly pricing scheme, a steeper unit rate and lower starting point price schedule is charged under APR. This generates larger variable profits for the monopolist while makes consumers worse off overall. A small number of low type consumers can purchase at a lower price leading to a smaller deadweight loss compared to the MPR (deadweight loss HKG smaller than IJG). However, most of the consumers are worse off for paying a higher price at B rather than at C due to the biased APR.

Once restricted by the profit constraint, the monopolist is forced to charged at a lower mark up ratios: Both price schedules under APR and MPR become flatter. Interestingly, the policy may induce a higher unit rate price schedule under MPR than under APR (since both optimal unit rate price schedules are increasing, the marginal rate should be above the average rate to allow the same generated variable profits). The welfare under APR schedule, hence, would be higher than under MPR (for smaller deadweight loss is arised).



(A) Deadweight loss in monopoly pricing



(B) Deadweight loss in regulated firm pricing

FIGURE 5: Deadweight loss in optimal monopoly pricing and regulated firm pricing.

Note: The horizontal axis is for quantity q, the vertical axis is for the responding (perceived) price p. Parameter values are $c = 1.5, \alpha = 3$. In monopoly pricing, $\mathcal{R}^m = 1 = \mathcal{R}^a$. In regulated firm pricing, $\mathcal{R}^m = 0.5, \mathcal{R}^a = 0.15$.

The binding profit constraint in regulated firm pricing makes both the optimal marginal price and average price schedules flatter. However, their slopes have changed at different speed, leading to the regulated marginal price schedule above the regulated average price in the end. Therefore, the deadweight loss under APR is smaller then under MPR, in regulated firm pricing scheme.

6 Conclusion

From an economic efficiency standpoint, increasing marginal price schedule for a monopolist and for a regulated firm is a puzzle since the literature of nonlinear pricing implies the optimal price schedule has decreasing marginal prices. However, recent evidence points out that electricity consumers respond to changes in average prices rather than to changes in marginal prices due to the complexity of nonlinear price schedules, see Ito (2014). We characterize optimal nonlinear prices in the world of APR consumers. We show that the optimal nonlinear price schedule may have increasing marginal prices, thereby implying that the increase block tariffs used by regulators may achieve the double goals of efficiency and equity.

In the world of APR, under price regulation, the new nonlinear price schedule may lead to a higher social welfare than the price schedule in the world of MPR. It should be noticed that the social welfare in the world of APR under unregulated firm pricing schedules is lower than in the world of MPR. This implies that consumers get worse off due to their biased APR and the monopolist can exploit it under unregulated prices. However, the regulated pricing can mitigate the deadweight loss due to the bias and improve efficiency.

A simulation to get the estimate of welfare improvements will be useful to be included.

Appendices

A Monopoly Pricing Under Marginal Price Response

Let the indirect utility from the IC constraint be $V(\theta) \equiv max_q U(q,\theta) - P^m(q)$. Following the standard literature results, the IC constraint requires that $Q^m(\theta)$ weakly increases in θ and that the marginal surplus $V_\theta > 0$. The IR constraint is satisfied if and only if $V(\theta_0) \geq 0$ (the utility of the lowest type is nonnegative), given that $V_\theta > 0$. The expected profits can be written in terms of expected total surplus minus consumer surplus:

$$\begin{aligned}
&\int_{\theta_0}^{\theta_1} (P(Q^m(\theta)) - C(Q^m(\theta)))dF(\theta) \\
=& \int_{\theta_0}^{\theta_1} (U(Q^m(\theta),\theta) - C(Q^m(\theta)) - V(\theta))dF(\theta) \\
=& \int_{\theta_0}^{\theta_1} \left((U(Q^m(\theta)),\theta) - C(Q^m(\theta)) - \frac{1-F(\theta)}{f(\theta)} \cdot U(Q^m(\theta),\theta) - V(\theta_0) \right) dF(\theta) \qquad (83)
\end{aligned}$$

The last equation is obtained by using integration by parts. Hence, the firm's pricing problem is transformed to the problem of choosing $Q^m(\theta)$ to maximize 83 subject to $Q(\theta)$ nondecreasing and $V(\theta_0) \geq 0$. The optimal price schedule is recovered from $U(Q^m(\theta),\theta) - P(Q(\theta)) = V(\theta)$, where $V(\theta) = V(\theta_0) + \int_{\theta_0}^{\theta_1} U_\theta(Q^m(\theta),\theta)d\theta$. Solving these gives the following results.

Result 1. *When the consumer responds to marginal price, the optimal consumption* $q^m = \max\{Q^m(\theta), 0\}$ *where* $q^m = Q^m(\theta)$ *satisfies:*

$$U_q - C_q = \frac{1-F}{f} \cdot U_{q\theta} \qquad (84)$$

The optimal price schedule $P^m(q)$ *satisfies:*

$$U_q(Q^m(\theta),\theta) - P^m_q(Q^m(\theta)) = 0 \qquad (85)$$

We also have

$$\begin{aligned}
Q^m_\theta \ &\textit{is nondecreasing} \qquad (86) \\
V(\theta_0) = U(Q^m(\theta_0),\theta_0) - P^m(Q(\theta_0)) &\geq 0 \qquad (87)
\end{aligned}$$

B Regulated Firm Pricing Under Marginal Price Response

Similar to the monopoly pricing framework, the optimization problem can be transformed into

$$\max_q \int_{\theta_0}^{\theta_1} U(q,\theta) - C(q)d\theta \tag{88}$$

$$\text{s.t.} \quad \int_{\theta_0}^{\theta_1} \left((U(Q^m(\theta)),\theta) - C(Q^m(\theta)) - \frac{1-F(\theta)}{f(\theta)} \cdot U(Q^m(\theta),\theta) - V(\theta_0) \right) dF(\theta) = F \tag{89}$$

$$q \text{ increasing in } \theta \tag{90}$$

$$U(q(\theta_0),\theta_0) \geq 0 \tag{91}$$

Using the Lagrangian function and its first order condition, we get

Result 2. *When the consumer responds to marginal price, the optimal consumption* $q^m = \max\{Q^m(\theta), 0\}$ *where* $q = Q^m(\theta)$ *satisfies:*

$$U_q - C_q = \left(\frac{\lambda}{1+\lambda}\right) \cdot \left(\frac{1-F}{f} \cdot U_{q\theta}\right) \tag{92}$$

The optimal price schedule $P^m(q)$ *satisfies:*

$$U_q(Q^m(\theta),\theta) - P^m_q(Q^m(\theta)) = 0 \tag{93}$$

We also have

$$Q^m_\theta \text{ is nondecreasing} \tag{94}$$

$$V(\theta_0) = U(Q^m(\theta_0),\theta_0) - P^m(Q(\theta_0)) \geq 0 \tag{95}$$

Notice this means

$$\frac{MP - MC}{MP} = \frac{\mathcal{R}}{\epsilon} \tag{96}$$

where $\mathcal{R} \in [0,1]$ is Ramsey number and ϵ is price elasticity of demand for qth unit across all types.

$$\epsilon = \frac{f}{1-F} \cdot \frac{U_q}{U_{q\theta}} \tag{97}$$

C Omitted Proofs

Demonstration of Corollary 2. Remind that under MP response, we have:

$$\frac{MP-MC}{AP}=\frac{\mathcal{R}^m}{\epsilon} \tag{98}$$

$$\Leftrightarrow 1-\frac{MC}{MP}=\frac{\mathcal{R}^m}{\epsilon} \tag{99}$$

Hence if ϵ is constant (and the cost is strictly convex) or increasing in q then the marginal price is decreasing. □

Demonstration of Corollary 3. We have

$$\frac{AP-MC}{AP}=\frac{\mathcal{R}^a}{\eta} \tag{100}$$

$$\Leftrightarrow 1-\frac{MC}{AP}=\frac{\mathcal{R}^a}{\eta} \tag{101}$$

Hence if η is constant (and the cost is strictly convex) or increasing in q then the average price is increasing. □

Demonstration of Corollary 4. When consumers respond to marginal price, the optimal pricing scheme satisfies

$$\frac{U_q-C_q}{U_q}=\mathcal{R}^m\cdot\frac{1-F}{f}\cdot\frac{U_{q\theta}}{U_q} \tag{102}$$

$$\Rightarrow \frac{MP-MC}{MP}=\frac{\mathcal{R}^m}{\epsilon}=\frac{\mathcal{R}^m}{\frac{f}{1-F}\cdot\theta} \tag{103}$$

Notice that $\frac{d\epsilon}{dq}=\frac{d\epsilon}{d\theta}\frac{d\theta}{dq}=(\frac{dh}{d\theta}\theta+h(\theta))\frac{d\theta}{dq}$ where $h(\theta)$ is the hazard rate $\frac{f}{1-F}$. Since we have $\frac{d\theta}{dq}>0$, the increasing hazard rate implies the increasing elasticity, thus decreasing marginal price, and decreasing average rate.

When consumers respond to average price, the optimal pricing scheme satisfies

$$\frac{U_q-C_q}{U_q}=\mathcal{R}^a\cdot\frac{-qU_{qq}}{U_q} \tag{104}$$

$$\Rightarrow \frac{AP-MC}{AP}=\mathcal{R}^a\cdot\frac{-qV_{qq}}{V_q} \tag{105}$$

Therefore, the increasing curvature implies the increasing average price. □

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