More Practice Questions for Test 2
May 25, 2016

- You should show your work for full credit in all problems in this exam.
- The followings are candidates that can be either a probability function or a probability density function for a random variable X. You may refer to the following functions when you solve the problems in this exam. Note that some of them are not valid at all. In addition, some of them have nothing to do with problems in this exam. Be careful when you refer to the following functions.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f(x) = \binom{n}{x} (p)^n (1-p)^{(x-n)} \qquad \text{for } x = 0, 1, 2, 3, ...n$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$$

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \qquad \text{for } x = 0, 1, 2, 3, 4, ...$$

$$f(x) = \binom{n}{x} (p)^x (1-p)^{(n-x)} \qquad \text{for } x = 0, 1, 2, 3, ...n$$

1.
We have a discrete random variable X.
The possible values that X can take are -2,-1,0,1, and 2. The probability function for X is given as follows.

$$f(x) = \begin{cases} \frac{x^2}{4} & \text{for } x = -1, 1 \\ \frac{1}{6} & \text{for } x = -2, 0, 2. \end{cases}$$

a. Is this probability function valid? Explain.

b. What is the probability that X is less than or equal to 0?

c. What is the mean and variance of X?

d. Calculate $E(2+3X)$, $Var(4+5X)$, and standard deviation of $4+5X$.

2.
Military radar and missile detection systems are designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a particular detection system has a .70 probability of detecting a missile attack. Use the binomial probability distribution to answer the following questions.

a. What is the probability that a single detection system will detect an attack?

b. If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack? (For full credit, write the probability function that you are using in this question and show your work.)

c. If three systems are installed, what is the probability that at least one of the systems will detect the attack? (For full credit, write the probability function that you are using in this question and show your work.)

3.
Suppose that a random variable X has the following probability density function.

$$f(x) = \begin{cases} \frac{1}{20} & \text{for } L \leq x \leq 140 \\ 0 & \text{otherwise} \end{cases}$$

That is, X follows the uniform distribution and its relevant range is from L to 140, where $L < 140$.

a. Draw the above probability density function.

b. What is the appropriate value for L? Explain how you found it.

c. Draw $f(x)$ and shade the area that represents the probability that X is less than 130. Then, calculate the probability that X is less than 130?

d. What is the probability that X is greater than 135 but less than 148?

4.
Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 3 passengers per minute.

a. Compute the probability of no arrivals in a **one-minute period.** (Explicitly write the probability function that you use in this question and show your work for full credit.)

b. Compute the probability of no arrivals in a **15-second period.** (Explicitly write the probability function that you use in this question and explain how you obtain that probability function.)

c. Compute the probability that the number of arrivals is greater than or equal to 3 in a **15-second period**. (Show your reasoning and work for full credit.)

5.
Suppose that a random variable X has the following probability density function.

$$f(x) = \frac{1}{50\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-365)^2}{50^2}\right)}$$

a. Considering the form of the probability density function, X follows a normal distribution. What is the mean and the variance of X?

b. Calculate $P[X \geq 400]$. (Show how you convert the normal distribution to the standard normal distribution for full credit. If you stuck, begin with drawing the graph of $f(x)$ and finding the area representing $P[X \geq 400]$.)

c. Calculate $P[X < 340 \; OR \; X > 400]$. (Show how you convert the normal distribution to the standard normal distribution for full credit. If you stuck, begin with drawing the graph of $f(x)$ and finding the area representing $P[X < 340 \; OR \; X > 400]$.)

d. Find the value V which makes $P[X \geq V]$ be equal to 0.025. (Show your reasoning and work for full credit.)