Name: Student ID:

BNAD/ECON/MGMT 276. STATISTICAL INFERENCE IN MANAGEMENT

Presession. Summer 2016.

TEST 2. DATE: 27 MAY 2016

Time: 75 minutes

NOTE:

1. There are total 9 questions. Answer all questions. The total points are 30.

2. You can use calculator. Your calculator may be programmable but you are not allowed to use the programmable tools.

3. You get partial credits if you can show the correct formula

4. Questions 6 and 9 are more difficult than others. You may prefer to do the other questions first.

Good luck!

Question 1. (2 points) You are given 8formulas/descriptions/notations on the left and 11 statistical concepts on the right column. Please match each formulas/description to itscorresponding statistical concepts. Notice there are **3 redundant concepts**, i.e. 3 concepts that do not have the corresponding descriptions.

Hence, you need to **match 8 formulas/descriptions to the correct 8 concepts**. Each correct match worths 0.25 point.

1. $f(x) = \frac{1}{b-a} \ \ for \ \ a \leq x \leq b$
2. $bell\ curve/bell\ shape$
3. $\sum xf(x)$
4. $E(X^2) - \mu^2$
5. $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$
6. $\sum x^2 f(x)$
7. σ
8. Number of occurrences in a time period

A. Bernoulli distribution
B. Binomial distribution
C. Poisson distribution
D. Normal distribution
E. Uniform distribution
F. Standard deviation
G. Sample variance
H. Mean
I. Sample mean
J. Variance
K. Second moment, $E(X^2)$

Please write down your answer as specific as 1A, 2B, 3C… below:

1.

2.

3.

4.

5.

6.

7.

8.

Question 2. (7 points). We have a discrete random variable X.

The possible values that X can take are -1, 0, 1, 2. The probability function of X is given as follows:

$$f(x) = \begin{cases} \dfrac{x^2}{10} \; for \;\; x \; = \; 2 \\ \dfrac{1}{5} \;\; for \;\; x = -1, 0, 1 \end{cases}$$

a. (1 point) Is this probability function valid? Explain.

b. (1 point) What is the probability that X is **less than or equal to 0**?

c. (1 point) What is the expected value of X?

d. (1 point) What is the variance of X?

Suppose that we construct a new random variable $Y = 2 + 3X$ based on the above random variable X.

e. (1 point) What is the **probability** that Y is 8?

f. (1 point)What is the mean of Y?

g. (1 point) What is the variance of Y?

Question 3.(4 points) Suppose that a random variable X has the uniform distribution with the support from 10 to 40,

$$f(x) = \begin{cases} \frac{1}{30} & for \quad 10 \leq x \leq 40 \\ 0 \ , & otherwise \end{cases}$$

a. (1 pt) Draw the above probability density function.

b. (1pt) What is the probability that X is from 10 to 20 (ie. greater than 10 but less than 20)?

c. (1 point) What is the probability that X is less than 15?

d. (1 point) What is the expected value of X?

Question 4. (4 points) Suppose customers arrive randomly and independently at a restaurant in Tucson. The receptionist notices that on average, there are 5 customers arriving in the restaurant **per hour**.

a. (1pt) What is the probability that there is **no customers** in the restaurant in an hour period?

b. (1 pt) What is the probability that there is **at least 2** customers arriving in the restaurant in an hour period?

c. (1 pt) Let Y be the number of customers arriving in the restaurant in the restaurant in a **30-minute period**. What is the distribution of Y? (Give the name of the distribution and the probability density function f(y) with clearly stating the expected value $E(Y) = \mu_Y = ?$)

d. (1 pt) What is the probability that there is **at least one** customer arriving in the restaurant in a **15-minute period**?

Question 5. (3 points) A university find that it is 20% likely that a student would enroll in the statistics course in their second year. Consider 10 random students in their second year. We are interested in the number of students that enrolls in the statistics course (in their second year).

a. (1 pt) What is the probability that there is **exactly one student** enrolling in the statistics course in their second year?

b. (1 pt) What is the probability that there is **at least 2 students** enrolling in the statistics course (in their second year)?

c. (1pt) What is the expected number of students enrolling in the statistics course (in their second year)?

Question 6. (1 point) Suppose that a random variable X has the uniform distribution with the support from 30 to H, where 30 < H

$$f(x) = \begin{cases} 0.2 & for \;\; 30 \le x \le H \\ 0 \;, & otherwise \end{cases}$$

What is the appropriate value for H? Explain how you found it.

Question 7 (2 points)

Suppose Z is standard normal random variable, and let σ be the standard deviation of Z.

a. (1 pt) What is the probability that P(Z < 1.25)?

b. (1pt) What is the probability that Z is between $-\sigma$ and σ ?

Question 8. (6 points) Suppose that a random variable X has the following probability density function:

$$f(x) = \frac{1}{20\sqrt{2\pi}} e^{\frac{-1}{2} \cdot \frac{(x-100)^2}{400}}$$

a. (1 pt) What is the name of the distribution above? What are the mean and the variance of X?

b. (1 pt) Draw the pdf $f(x)$above. Put x on the horizontal axis and $f(x)$ on the vertical axis. Indicate clearly the **location of the mean**. (You don't need to draw a very precise graph. You only need to show how it looks like and the location of the mean on the graph)

c. (1 pt) Calculate $\mathrm{P}(\mathrm{X} < 140.2)$. Show your work, not just the final number.

d. (1 pt) Calculate $\mathrm{P}(60.8 < X < 140.2)$. Show your work, not just the final number.

e. (1 pt) What is the probability that $P(X > 138.2 \;\text{ or }\; X < 124.2)$?

f. (1 pt) Find the value V such that the likelihood that X is greater than V is 12.3%

Question 9.(1 point) There is a mathematical operation called Integration. To denote this operation, the symbol $\int$ is used and it is called the Integral. What this operation does is that it gives some area under a function. For example, if we calculate $\int_a^b f(x)dx$, it merely gives us the area under $f(x)$ and between a and b. Knowing this, calculate the following expression:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{50\pi}} e^{\frac{-(x-3)^2}{50}} dx$$