

BNAD/ECON/MGMT 276. STATISTICAL INFERENCE IN MANAGEMENT**Pre-session. Summer 2017.****TEST 2. DATE: 26 MAY 2017****Time: 75 minutes**

NOTE:

1. There are total 8 questions for a possible total of 30 points. Questions 9 and 10 are extra credit questions.
2. You can use calculator. Your calculator may be programmable but you are not allowed to use the programmable tools.
3. You get partial credits if you can show the correct formula in a meaningful mathematical expression.
4. Questions 6 and 9 are more difficult than others. You may prefer to do the other questions first.
5. I will not answer any questions related to the content of the questions. Do your best.

Good luck!

Question 1. (2 points) You are given 8 formulas/descriptions/notations on the left and 11 statistical concepts on the right column. Please match each formulas/description to its corresponding statistical concepts. Notice there are **3 redundant concepts**, i.e. 3 concepts that do not have the corresponding descriptions.

Hence, you need to **match 8 formulas/descriptions to the correct 8 concepts**. Each correct match is worth 0.25 point.

- | | |
|---|----------------------------|
| 1. $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$ | A. Bernoulli distribution |
| 2. <i>bell curve/bell shape</i> | B. Binomial distribution |
| 3. $\sum xf(x)$ | C. Poisson distribution |
| 4. $E(X^2) - \mu^2$ | D. Normal distribution |
| 5. $f(x) = \binom{n}{x}p^x(1-p)^{n-x}$ | E. Uniform distribution |
| 6. $\sum x^2f(x)$ | F. Standard deviation |
| 7. σ | G. Sample variance |
| 8. Number of occurrences in a time period | H. Variance |
| | I. Sample mean |
| | J. Mean |
| | K. Second moment, $E(X^2)$ |

Please write down your answer (by simply listing the relevant letter) below:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.

Question 2. (7 points). We have a discrete random variable X .

The possible values that X can take are $-1, 0, 1, 2$. The probability function of X is given as follows:

x	$f(x)$
-1	0.1
0	0.2
1	0.3
2	0.4

a. (1 point) Is this probability function valid? Explain.

b. (1 point) What is the probability that X is **less than or equal to 0**?

c. (1 point) What is the expected value of X ?

d. (1 point) What is the variance of X ?

Suppose that we construct a new random variable $Y = 2 + 3X$ based on the above random variable X .

e. (1 point) What is the **probability** that Y is 8?

f. (1 point) What is the mean of Y ?

g. (1 point) What is the variance of Y ?

Question 3. (4 points) Suppose that a random variable X has the uniform distribution with the support from 10 to 40,

$$f(x) = \begin{cases} \frac{1}{30} & \text{for } 10 \leq x \leq 40 \\ 0 & , \quad \text{otherwise} \end{cases}$$

a. (1pt) Draw the above probability density function.

b. (1pt) What is the probability that X is from 10 to 20 (ie. greater than 10 but less than 20)?

c. (1 point) What is the probability that X is less than 15?

d. (1 point) What is the expected value of X ?

Question 4. (4 points) Suppose customers arrive randomly and independently at a restaurant in Tucson. The receptionist notices that on average, there are 5 customers visiting the restaurant **per hour**.

a. (1pt) What is the probability that there is **no customers** in the restaurant in an hour period?

b. (1 pt) What is the probability that there is **at least 2** customers visiting the restaurant in an hour period?

- c. (1 pt) Let Y be the number of customers arriving in the restaurant in a **30-minute period**. What is the distribution of Y ? Specifically, give the name of the distribution and the probability function $f(y)$ with clearly stating the expected value $E(Y) = \mu_Y = ?$

- d. (1 pt) What is the probability that there is **at least one** customer arriving in the restaurant in a **15-minute period**?

Question 6. (1 point) Suppose that a random variable X has the uniform distribution with the support from 30 to H , where $30 < H$

$$f(x) = \begin{cases} 0.2 & \text{for } 30 \leq x \leq H \\ 0 & , \quad \text{otherwise} \end{cases}$$

What is the appropriate value for H ? Explain how you found it.

Question 9. (4 points) Suppose that a random variable X follows a normal distribution with mean of 100 and the standard deviation of 20.

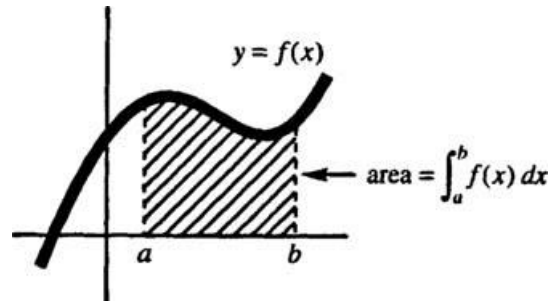
a. (1 pt) Calculate $P(X < 140.2)$. Show your work, not just the final number.

b. (1 pt) Calculate $P(60.8 < X < 140.2)$. Show your work, not just the final number.

c. (1 pt) What is the probability that $P(X < 124.2 \text{ or } X > 138.2)$?

d. (1 pt) Find the value V such that the likelihood that X is greater than V is 12.3%

Question 9. (extra credit question, 2 pts) There is a mathematical operation called Integration. To denote this operation, the symbol \int is used and it is called the Integral. What this operation does is that it measures the area under a function. For example, if we calculate $\int_a^b f(x) dx$, it gives us the area under $f(x)$ and between vertical lines a and b .



Knowing that, calculate the following expression:

$$\int_{-\infty}^3 \frac{1}{\sqrt{50\pi}} e^{-\frac{(x-3)^2}{50}} dx$$

Question 10. (Extra credit question, 1 pt) Consider the following experiment: roll a die 5 times. Each time, the die is rolled in an identical way. Assume the die is fair, i.e. each outcome equally likely happens. We are interested in the number that appears in each outcome. Is this experiment a binomial experiment? You have to explain your answer to get credit.