1.

We have a discrete random variable X.

The possible values that X can take are -2,-1,0,1, and 2. The probability function for X is given as follows.

$$f(x) = \begin{cases} \frac{x^2}{4} & \text{for } x = -1, 1 \\ \frac{1}{6} & \text{for } x = -2, 0, 2. \end{cases}$$

a. Is this probability function valid? Explain.

b. What is the probability that X is less than or equal to 0?

c. What is the mean and variance of X?

c. Calculate $E(2+3X)$, $Var(4+5X)$, and standard deviation of $4+5X$.

2.

Military radar and missile detection systems are designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a detection system has a .70 probability of detecting a missile attack. The ministry is considering to install more systems. Assume the systems will be working identically and independently.

a. If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack?

b. If three systems are installed, what is the probability that at least one of the systems will detect the attack?

3.

Suppose that a random variable X has the following probability density function.

$$f(x) = \begin{cases} \frac{1}{20} & \text{for } L \leq x \leq 140 \\ 0 & \text{otherwise} \end{cases}$$

That is, X follows the uniform distribution and its relevant range is from L to 140, where $L < 140$.

a. Draw the above probability density function.

b. What is the appropriate value for L? Explain how you found it.

4.

Suppose that a random variable X follows the uniform distribution and its support is from 100 to 130.

    a. Draw the above probability density function.

    b. Calculate the probability that X is less than 120?

    c. What is the probability that X is greater than 110 but less than 120?

5.
Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 3 passengers per minute.

   a. Calculate the probability of **no arrivals** in a **one-minute period.**

   b. Calculate the probability of **no arrivals** in a **15-second period.**

   c. Compute the probability that there is **at least one** arriving passenger in a **15-second period**. (Show your reasoning and work for full credit.)

6.

Suppose that a random variable X has the following probability density function.

$$f(x) = \frac{1}{50\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-365)^2}{50^2}\right)}$$

  a. What is the name of the above distribution? What is the mean and the variance of X?

  b. Calculate $P(X \leq 350)$.

  c. Calculate $P(X \geq 330)$.

d. Calculate $P(340 < X < 400)$.

e. Calculate $P(X < 360 \; OR \; X > 410)$.

f. Find the value V which makes $P(X \geq V)$ be equal to 0.025.